Multi-type Resource Allocation with Partial Preferences

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Multi-type Resource Allocation (MTRA)

\[ \pi \text{ agents} \]
\[ \sigma \text{ types of items} \] (n of each type)
\[ \begin{array}{c}
\text{S :} \\
\text{C :}
\end{array} \]

Assignments: Each agents' allocation is a collection of fractional bundles

Partial Preferences

\[ \text{Acyclic CP-net} \rightarrow \text{Partial Preference} \]

Compute an assignment that is fair and economically efficient

**Upper Contour Set** @ bundle \( x \) w.r.t. \( > \) is \( U(\succ, x) = \{y : y \succ x \text{ or } y = x\} \)

Allocation \( p \) stochastically dominates \( q \) w.r.t. \( > \), if @ every bundle \( x \),
\[ \Sigma_{y \in U(\succ, x)} p_y \geq \Sigma_{y \in U(\succ, x)} q_y \]

Assignment \( P \) stochastically dominates \( Q \) if \( p_j \succ^d q_j \text{ for every agent } j \)

Fairness, Efficiency, Strategyproofness, and Indivisibility

A fractional assignment \( P \) satisfies:
- decomposability: it is a probability distribution over discrete assignments
- equal treatment of equals: agents with identical preferences receive identical allocations
- sd-envy-freeness: if for every pair of agents \( j, j', P_j >^d P_{j'} \)
- sd-efficiency: no assignment \( Q \) s.t. for every agent \( j, Q_j >^d P_j \)
- ordinal fairness: if for every pair of agents \( j, j' \), and every bundle \( x \), s.t. \( P_j x > 0 \), we have \( \Sigma_{y \in U(\succ, x)} P_j y \leq \Sigma_{y \in U(\succ, x)} Q_j y \)
- weak-sd-envy-freeness: if for every pair of agents \( j, j' \), \( P_j >^d P_{j'} \Rightarrow P_j = P_{j'} \)
- ex-post-efficiency: a probability distribution over sd-efficient discrete assignments

A mechanism \( f \) satisfies:
- sd-strategyproofness: if \( f(\succ) >^d f(\succ') \), for every agent \( j \), every misreport
- sd-weak-strategyproofness: if \( f(\succ') >^d f(\succ) \Rightarrow f(\succ') = f(\succ') \)

In general, mechanism \( f \) satisfies property \( X \) if \( f(\succ) \) satisfies \( X \) for every profile \( \succ \)

Assignments are NOT guaranteed to be decomposable when there are multiple types of items

NO mechanism is sd-efficient AND sd-envy-free under general partial preferences

Multi-type Random Priority (MRP)

Extends the Random Priority (RP) mechanism [Abdulkadioglu and Sonmez, 1998]
- Topologically sort partial order \( \succ_j \) to linear order \( \succ_j' \)
- Pick priority ordering \( \sigma \) over agents uniformly at random
- Agents arrive according to \( \sigma \), and are allocated their favorite remaining bundle
- Remove agent and all items in bundle

Multi-type Probabilistic Serial (MPS)

Extends Probabilistic Serial (PS) mechanism [Bogomolnaia and Moulin, 2001]
- Topologically sort partial order \( \succ_j \) to linear order \( \succ_j' \)
- While there is a remaining item:
- ALL agents simultaneously consume their favorite remaining bundle (per \( \succ_j' \)) at an equal, uniform rate until supply of any item being consumed is exhausted

Multi-type General Dictatorship (MDG)

Hybrid of MRP and MPS
- Topologically sort partial order \( \succ_j \) to linear order \( \succ_j' \)
- For \( j = 1, \ldots, \pi \) do:
- Agent \( j \) invites all other agents \( j' \) s.t. \( \succ_j'^{j} \) to simultaneously consume their favorite remaining bundle until some item being consumed is exhausted

Fairness, Efficiency, and Non-Manipulability

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<th>PE</th>
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Results annotated with $\dagger$ are due to [Bogomolnaia and Moulin, 2001], and those annotated $\ddagger$ are due to [Hashimoto et al., 2014]

Future Work

- Characterizing MRP and MPS
- Stronger properties under natural restrictions on the domain of preferences

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