Design and Analysis of Algorithms

Homework Assignment 3

Due: February 22 in class

Please include your full name on all homework assignments. Make sure that the assignments are clear and easy to evaluate. Homework is graded at the discretion of the grader.

1. An \( m \times n \) Young tableau is an \( m \times n \) matrix such that the entries of each row are in sorted order from left to right and the entries of each column are in sorted order from top to bottom. Some of the entries of a Young tableau may be \( \infty \), which are treated as non-existent elements. Thus, a Young tableau can be used to hold \( r \leq m n \) finite elements. [40%]

   a) a) Draw a 4x4 Young tableau containing the elements \{9,16,3,2,4,8,5,14,12\}

   b) b) Give an algorithm to implement DELETE-MIN operation (return the minimum element of a Young tableau and delete it) on a non-empty \( m \times n \) Young tableau that runs in \( O(n+m) \) time. Your algorithm should use a recursive subroutine that solves an \( m \times n \) problem by recursively solving either an \( (m-1) \times n \) or an \( m \times (n-1) \) subproblem.

   c) c) Show how to insert a new element into a non full \( m \times n \) Young tableau in \( O(m+n) \) time.

2. Show the binomial heaps generated by the following sequence of inserts from left to right: 11, 28, 5, 7, 32, 18, 65, 9, 43. You should start with an empty binomial heap and show the binomial heap after 11 is inserted, 28, etc. [20%]

3. Explain how the heap property is maintained when new elements are inserted into a binomial heap. [10%]

4. Given sets \{1,7,10\} and \{4,15\} [30%]

   a. Show the backward tree representation of the sets.
   b. Show the single array that would be used to represent the two sets in question one.
   c. Give the backward tree after the two sets are merged into a single set
   d. Give the array that represent the new graph produced in the previous question
   e. Show the compressed tree after find(5) is executed.