



# Subspace morphing theory for appearance based object identification

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## Abstract

Object identification techniques have wide applications ranging from industry, business, military, law enforcement, to people's daily life. This research is motivated to develop a new theory for appearance based object identification with its applications in different areas. Although many successful techniques have been proposed in certain specific applications, object identification, in general, still remains as a difficult and challenging problem. In appearance based approaches, almost all the proposed methods are based on a fundamental assumption, i.e., all the images (both in the model base and to be queried) are in the same dimensions, so that the feature vectors are all in the same feature space; if images are provided with different dimensions, a normalization in scale to a pre-determined image space must be conducted. In this research, a theory for appearance based object identification called subspace morphing is developed, which allows scale-invariant identification of images of objects, and therefore, does not require normalization. Theoretical analysis and experimental evaluation show that in the situation where images are provided in different dimensions, which is common in many applications, subspace morphing theory is superior to the existing, normalization-based techniques in performance. © 2002 Published by Elsevier Science Ltd on behalf of Pattern Recognition Society.

**Keywords:** Appearance based object identification; Subspace morphing; SV vector; Projections; Morphed vectors; Essential SV vectors; Collection matrix; Essential collection matrix; Identification capability; Identification precision

## 1. Introduction

Object identification techniques have wide applications ranging from industry, business, military, law enforcement, to people's daily life. This research is motivated to develop a new theory for appearance based object identification with its applications in different areas.

Although many successful techniques have been proposed in certain specific applications, object identification, in general, still remains as a difficult and challenging problem. In appearance based approaches, typical methods include eigenfaces and the related subspace analysis (e.g. [1-4]),

singular value decomposition (e.g. [5]), Gabor wavelet features (e.g. [6]), neural networks based learning (e.g. [7,8]), evolutionary pursuit based learning (noticeably [9,10]), on active appearance models (noticeably [11,12]), and shape and appearance-based probabilistic models (e.g. [13-15]) as well as the combinations of several existing techniques in object identification (e.g., matrix distances proposed by Cox et al. [16]). All these methods are based on a fundamental assumption, i.e., all the images (both in the model base and to be queried) are in the same dimensions, so that the feature vectors are all in the same feature space; if images are provided with different dimensions, a normalization in scale to a pre-determined space must be conducted.

In this paper, a theory for appearance based object identification called *subspace morphing* is developed, which allows scale-invariant identification of images of objects, and therefore, does not require normalization in scale.

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Theoretical analysis and experimental evaluation show that in the situation where images are provided in different dimensions, which is typical and common in many applications, subspace morphing theory is superior to the existing, normalization-based techniques in performance.

This paper is organized as follows. After this section, the subspace morphing theory is introduced in Section 2. The following section is dedicated to comparing this theory with the existing, normalization based techniques in terms of computation complexity, identification capability, and identification precision. Then the experimental evaluations of this theory and the normalization based techniques are conducted based on real image data. Finally, a conclusion is presented.

## 2. Subspace morphing theory

In the following text,  $R$  is used to denote a real space; thus,  $R^n$  is an  $n$  dimensional vector space, and  $R^{m \times n}$  is an  $m \times n$  dimensional matrix space (or image space). Also a boldface symbol is used to denote a vector or matrix.

The subspace morphing theory is based on a set of properties of the theory of Singular Value Decomposition [17].

**Theorem 1.** Let  $A \in R^{m \times n}$ .  $\exists$  column-orthonormal matrix  $U \in R^{m \times n}$ , row-orthonormal matrix  $V \in R^{n \times n}$ , and diagonal matrix  $\Lambda \in R^{n \times n}$ , such that

$$A = U\Lambda V^T, \quad (1)$$

where the diagonal values of  $\Lambda$  are in sorted order from the largest to the smallest.

**Proof.** See Ref. [17].

**Definition 1.** Let  $A \in R^{m \times n}$ . Eq. (1) defines a *singular value decomposition* (SVD) of the matrix  $A$ , where the diagonal values of  $\Lambda$  are called *singular values* (SVs). The vector formed by the sorted diagonal values are called *SV vectors*.

**Lemma 1.** Let  $A \in R^{m \times n}$ . Let  $P \in R^{m \times m}$ , and  $Q \in R^{n \times n}$  be orthonormal matrices. Then matrices  $PA$ ,  $AQ$  and matrix  $A$  share the same set of singular values.

**Proof.** W.L.O.G. assume  $m > n$ . Then, by SVD, Eq. (1) holds for  $A$ .

This is followed by

$$(PA)^T(PA) = A^T A = V A^2 V^T, \quad (2)$$

$$(AQ)^T(AQ) = Q^T(A^T A)Q = W A^2 W^T, \quad (3)$$

where  $W = Q^T V$  is an orthonormal matrix, as the product of two orthonormal matrices is another orthonormal matrix [18]. This has proved the lemma.  $\square$

**Lemma 2.** Let  $A \in R^{m \times n}$ . Let  $B \in R^{m \times n}$ , and  $B$  is obtained by the transform of interchanging any two rows (or columns) of  $A$ . Then  $A$  and  $B$  share the same set of singular values. 41 43

**Proof.** Define the following matrix: 45

$$I_{ij} = I - (E_i - E_j)(E_i - E_j)^T, \quad (4)$$

where  $I$  is the unit matrix, and  $E_i$  and  $E_j$  are the  $i$ th and the  $j$ th column vectors of the unit matrix, respectively. Hence, the transform of interchanging two rows of  $A$  is equivalent to the product of  $I_{ij}A$ , and the transform of interchanging two columns of  $A$  is equivalent to the product of  $AI_{ij}$ . It is clear that the matrix  $I_{ij}$  is an orthonormal matrix. Consequently, the proof follows by Lemma 1. 47 49 51

**Theorem 2.** Let  $A, B \in R^{m \times n}$ . By SVD,  $\exists$  SV vectors  $\Lambda$  and  $\Sigma$  for  $A$  and  $B$ , respectively. Thus, the Euclidean norm of the difference between the vectors  $\Lambda$  and  $\Sigma$  is bounded by the Frobenius norm of the difference between the matrices  $A$  and  $B$ , i.e. 53 55 57

$$\sqrt{\sum_{i=1}^n (\lambda_i - \sigma_i)^2} \leq \sqrt{\sum_{i=1}^m \sum_{j=1}^n (a_{ij} - b_{ij})^2}, \quad (5)$$

where  $\lambda_i, \sigma_i$  are the  $i$ th elements of  $\Lambda$  and  $\Sigma$ , respectively, and  $a_{ij}$  and  $b_{ij}$  are the elements of  $A$  and  $B$  at the position of  $(i, j)$ , respectively. 59

**Proof.** See [17]. 61

**Lemma 3.** Let  $A \in R^{m \times n}$ . Let  $B \in R^{p \times q}$ , such that  $B$  is obtained by scaling  $A$  in row by  $k_r$ , and then in column by  $k_c$ , where  $k_r, k_c$  are integers, and  $k_r, k_c \geq 1$ , i.e.,  $p = k_r m$ , and  $q = k_c n$ . Thus,  $B$  has the same rank as  $A$  does, and the singular values of  $B$  are  $\sqrt{k_r k_c}$  times the corresponding singular values of  $A$ . 63 65 67

**Proof.** The fact that  $B$  has the same rank as  $A$  does is obvious, since duplication of any row or column in a matrix does not affect the rank of the matrix [18]. 69

By SVD following the notations of Eq. (1): 71

$$A = U\Sigma V^T. \quad (6)$$

In the following, the symbol  $\equiv$  is used to denote “equal by definition”, and the symbol  $\rightarrow$  is used to denote a transform. 73

To show the relationship of their corresponding singular values between  $A$  and  $B$ , let 75

$$A \equiv \begin{pmatrix} \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_m^T \end{pmatrix},$$

where  $\mathbf{a}_1, \dots, \mathbf{a}_m$  are the row vectors of  $A$ .

1 The following series of transforms are conducted:

$$\begin{aligned}
 \mathbf{A} &= \begin{pmatrix} \mathbf{a}_1^\top \\ \vdots \\ \mathbf{a}_m^\top \end{pmatrix} \\
 &\rightarrow \begin{pmatrix} \left. \begin{matrix} \mathbf{a}_1^\top \\ \vdots \\ \mathbf{a}_1^\top \end{matrix} \right\} k_r \text{ times} \\ \vdots \\ \left. \begin{matrix} \mathbf{a}_m^\top \\ \vdots \\ \mathbf{a}_m^\top \end{matrix} \right\} k_r \text{ times} \end{pmatrix} \\
 &\equiv \mathbf{A}_r \\
 &\rightarrow \begin{pmatrix} \mathbf{A} \\ \vdots \\ \mathbf{A} \end{pmatrix} k_r \text{ times} \\
 &\equiv \mathbf{A}_b.
 \end{aligned}$$

3 The first transform duplicates each row of  $\mathbf{A}$  by  $k_r$  times  
 5 to obtain  $\mathbf{A}_r$ , and the second transform “clusters” the row  
 7 vectors of  $k_r \mathbf{A}$  by interchanging the positions of the row  
 9 vectors to obtain  $\mathbf{A}_b$ . Hence,

$$\begin{aligned}
 \mathbf{A}_b^\top \mathbf{A}_b &= (\mathbf{A}^\top \dots \mathbf{A}^\top) \begin{pmatrix} \mathbf{A} \\ \vdots \\ \mathbf{A} \end{pmatrix} \\
 &= \sum_{i=1}^{k_r} \mathbf{A}^\top \mathbf{A} \\
 &= k_r \mathbf{A}^\top \mathbf{A} \\
 &= k_r \mathbf{V} \boldsymbol{\Sigma}^2 \mathbf{V}^\top \\
 &= \mathbf{V} (\sqrt{k_r} \boldsymbol{\Sigma})^2 \mathbf{V}^\top.
 \end{aligned}$$

7 This means that the singular values of  $\mathbf{A}_b$  are  $\sqrt{k_r}$  times  
 9 those of  $\mathbf{A}$ . By Lemma 2,  $\mathbf{A}_b$  and  $\mathbf{A}_r$  have the same singular  
 11 values. Therefore, the singular values of  $\mathbf{A}_r$  are  $\sqrt{k_r}$  times  
 13 those of  $\mathbf{A}$ .

Similarly, the transform on  $\mathbf{A}_r$  by duplicating each  
 11 columns of  $k_c$  times to obtain a matrix  $\mathbf{A}_c$ , which is  $\mathbf{B}$ ,  
 13 and then by “clustering” the columns of  $k_c \mathbf{A}_r$  through in-  
 15 terchanging the positions of the columns to obtain a matrix  
 $\mathbf{A}_d$ . Then,

$$\begin{aligned}
 \mathbf{A}_d \mathbf{A}_d^\top &= (\mathbf{A}_r \dots \mathbf{A}_r) \begin{pmatrix} \mathbf{A}_r^\top \\ \vdots \\ \mathbf{A}_r^\top \end{pmatrix} \\
 &= k_c \mathbf{A}_r \mathbf{A}_r^\top.
 \end{aligned}$$

15 This means that the singular values of  $\mathbf{A}_d$  are  $\sqrt{k_c}$  times  
 those of the corresponding singular values of  $\mathbf{A}_r$ , which

17 have been shown to be  $\sqrt{k_r}$  times the corresponding singular  
 19 values of  $\mathbf{A}$ . Again, by Lemma 2,  $\mathbf{A}_d$  and  $\mathbf{A}_c$  have the same  
 21 singular values. Therefore, the singular values of  $\mathbf{A}_c$  are  
 $\sqrt{k_r k_c}$  times those of  $\mathbf{A}$ . This concludes the proof of this  
 Lemma.  $\square$

Based on the above lemmas and Theorem 2, the founda-  
 23 tion of the subspace morphing theory and its algorithm is  
 based on the following theorem:

**Theorem 3.** Let  $\mathbf{A}_1 \in R^{m_1 \times n_1}$ ,  $\mathbf{A}_2 \in R^{m_2 \times n_2}$ .  $\mathbf{A}$  and  $\boldsymbol{\Sigma}$  are  
 25 SV vectors of  $\mathbf{A}_1$  and  $\mathbf{A}_2$ , respectively. For any positive  
 27 integer  $n$ , the following holds:

$$\|\sqrt{m_2 n_2} \mathbf{A} - \sqrt{m_1 n_1} \boldsymbol{\Sigma}\|_n \leq \|\mathbf{B}_1 - \mathbf{B}_2\|_F, \quad (7)$$

where  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are both in  $R^{m_1 m_2 \times n_1 n_2}$ , and are obtained by  
 29 zero-hold interpolation [19] from  $\mathbf{A}_1$  and  $\mathbf{A}_2$ , respectively.  
 $\|\cdot\|_F$  stands for matrix Frobenius norm, and  $\|\cdot\|_n$  the  
 31 norm of an  $n$  dimensional vector.

**Proof.** First scale up matrices  $\mathbf{A}_1$  and  $\mathbf{A}_2$ , respectively, to  
 33  $\mathbf{B}_1$  and  $\mathbf{B}_2$ , such that  $\mathbf{B}_1, \mathbf{B}_2 \in R^{m_1 m_2 \times n_1 n_2}$ . By Lemma 3, the  
 singular value vectors of  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are:

$$\sqrt{m_2 m_2} \lambda_i, \quad i = 1, \dots, n_1$$

and

$$\sqrt{m_1 n_1} \sigma_i, \quad i = 1, \dots, n_2.$$

By Theorem 2, Eq. (7) holds, and the proof follows.

This foundation theorem may be explained conceptually  
 37 as follows. Given two images with different dimensions, they  
 39 can always be mapped onto a common image space. This  
 theorem establishes a relationship between the SV vectors  
 41 extracted from these two images and the SV vectors that  
 would be extracted after mapping them onto that common  
 43 space. Thus, using this theorem, the SV vectors from the  
 hypothetical common space can be immediately “extracted”  
 45 based on the SV vectors obtained from the current images  
 without actually mapping them onto that common space.

Based on this foundation theorem, the subspace morphing  
 47 theory is introduced below.

**Definition 2.** Let  $\mathbf{v}$  be a vector in  $R^n$ . Let  $R^m$  be an arbitrary  
 49 space. Define vector  $\mathbf{v}'$  as a *projection* of  $\mathbf{v}$  from  $R^n$  to  $R^m$  as  
 follows. If  $m > n$ , the dimensionality of  $\mathbf{v}'$  is extended to  $m$   
 51 by filling values 0 to the extended dimensions of  $\mathbf{v}$ ; if  $m < n$ ,  
 the dimensionality of  $\mathbf{v}'$  is truncated to  $m$ ; if  $m = n$ ,  $\mathbf{v}' = \mathbf{v}$ . 53

**Definition 3.** Given two arbitrary spaces  $R^{m \times n}$  and  $R^{p \times q}$ ,  
 W.L.O.G. assume  $m \geq n$ ,  $p \geq q$ . Let  $\mathbf{A} \in R^{m \times n}$ ,  $\mathbf{v}$  be its SV  
 55 vector, and  $\mathbf{u}'$  be the projection of  $\mathbf{v}$  from  $R^n$  to  $R^q$ . Vector  
 $\mathbf{v}'$  is defined as the *morphed SV vector* of  $\mathbf{v}$  from  $R^n$  to  $R^q$  57

1 as follows:

$$2 \quad \mathbf{v}' = \frac{\sqrt{pq}}{\sqrt{mn}} \mathbf{v}, \quad \mathbf{u}' = \sqrt{pq} \mathbf{u}, \quad (8)$$

where  $\mathbf{u} = (1/\sqrt{mn})\mathbf{v}'$  is called the *essential SV vector* of  $\mathbf{v}$ .

3 This means that  $\mathbf{v}'$  is the “equivalent” SV vector of a  
4 matrix in  $R^{p \times q}$  mapped from  $\mathbf{A}$  in  $R^{m \times n}$ . Note that the values  
5 of an essential SV vector always stay the same regardless  
6 of whatever space the corresponding vector is morphed to.

7 **Definition 4.** Let  $\mathbf{A} \in R^{m_1 \times n_1}$ ,  $\mathbf{B} \in R^{m_2 \times n_2}$ , W.L.O.G.  
8 assume  $m_1 \geq n_1$ ,  $m_2 \geq n_2$ . Let  $\mathbf{v}_A$  and  $\mathbf{v}_B$  be their SV  
9 vectors, respectively. Given any  $R^{p \times q}$ , W.L.O.G. assume  
10  $p \geq q$ . The *addition* of  $\mathbf{v}_A$  and  $\mathbf{v}_B$  in  $R^q$  is defined as

$$11 \quad (\mathbf{v}_A \oplus \mathbf{v}_B)_q = (\mathbf{v}'_A + \mathbf{v}'_B) = \sqrt{pq}(\mathbf{u}_A + \mathbf{u}_B), \quad (9)$$

12 where  $\mathbf{u}_A$  and  $\mathbf{u}_B$  are the essential SV vectors of  $\mathbf{v}_A$  and  $\mathbf{v}_B$ ,  
13 and  $\mathbf{v}'_A$  and  $\mathbf{v}'_B$  are the morphed SV vectors of  $\mathbf{v}_A$  and  $\mathbf{v}_B$ ,  
14 respectively.

15 Similarly, subtraction, dot product, and cross product of  
16 two SV vectors with different dimensionalities may be de-  
17 fined. Note that all these algebraic operations between two  
18 SV vectors are defined as to first morphing them to a “com-  
19 mon” space before the actual operation may be conducted;  
20 this “common” space may be any arbitrary space; this space  
21 is to be determined dynamically during the identification  
22 process.

23 **Definition 5.** Given a set of SV vectors:  $\mathbf{v}_i \in R^{n_i}$ , respec-  
24 tively,  $i = 1, \dots, k$ , let  $\mathbf{v}'_i$  be the morphed vector of  $\mathbf{v}_i$  from  $R^{n_i}$   
25 to a “common” space  $R^q$ , respectively. The matrix  $\mathbf{V}$  formed  
26 by all  $\mathbf{v}'_i$  is called the *collection matrix* of  $\mathbf{v}_i$  in  $R^q$ , i.e.

$$27 \quad \mathbf{V} = (\mathbf{v}'_1, \dots, \mathbf{v}'_k) = \sqrt{pq} \mathbf{U}, \quad (10)$$

28 where  $\mathbf{U}$  is called the *essential collection matrix* of all the  
29 vectors  $\mathbf{v}_i$  ( $i = 1, \dots, k$ ). Let  $\mathbf{u}_i$  be the essential SV vector of  
30  $\mathbf{v}_i$ . Then,  $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_k)$ .

31 Note that the essential collection matrix is independent  
32 of the “common” space that the given SV vectors are mor-  
33 phed to, which is to be determined dynamically during the  
34 identification process.

35 Based on the above theory, the algorithm (**SM**) for object  
36 identification is given below. Assume that there are  $N$  object  
37 classes in a model base, and each class has  $M_i$  sample images  
38 collected ( $i = 1, \dots, N$ ). Note that sample images in each  
39 class may be in different dimensions. Consequently, the SV  
40 vectors of these samples are in different dimensionalities,  
41 and so is the SV vector of the query image. The idea of **SM**  
42 is to use nearest linear combination *in a morphed subspace*  
43 dynamically, as opposed to in a fixed, predetermined space,  
44 to find the best match for a query image. Specifically, assume

45 that each sample image in a model base is represented by  
46 an essential SV vector  $\mathbf{u}_{ij}$  (here  $i$  indexes the class number,  
47 and  $j$  indexes the sample number in the object class). Then  
48 a linear combination  $\mathbf{U}_i \boldsymbol{\alpha}_i$  of all the essential SV vectors in  
49 class  $i$  is used to represent the “cluster region” of this class in  
50 a morphed feature space, where  $\mathbf{U}_i$  is the essential collection  
51 matrix of the SV vectors in class  $i$ , and  $\boldsymbol{\alpha}_i$  is a coefficient  
52 vector in  $R^{M_i}$ . Assume that  $\mathbf{w}$  is the essential SV vector of  
53 the query image. Then, the following constraint holds for a  
54 specific  $\boldsymbol{\alpha}_i$ , if the query image is in class  $i$ :

$$55 \quad \mathbf{w} = \mathbf{U}_i \boldsymbol{\alpha}_i. \quad (11)$$

56 Eq. (11) shows that a linear solution exists for the  
57 object identification problem *independent* of the hypotheti-  
58 cal “common” image space  $R^{p \times q}$  and the morphed subspace  
59  $R^q$ . This property enables vector space comparison in differ-  
60 ent dimensionalities independent of the hypothetical “com-  
61 mon” space. Eq. (11) is typically overdetermined, and a so-  
62 lution to  $\boldsymbol{\alpha}_i$  may be obtained through a standard least square  
63 minimization.

### 64 3. Subspace morphing vs. normalization 61

65 This section is dedicated to comparing the performance  
66 of the subspace morphing theory with those of the normal-  
67 ization based techniques, and then to show that the former  
68 is superior to the latter in performance. In particular, the  
69 performance analysis is conducted in the following three  
70 aspects:

71 *Computation complexity:* It is clear that the process of  
72 normalization in scale *per se* requires extra computation. *In*  
73 *addition* to this extra computation, in the case of scaling  
74 up, redundancy is introduced, as data interpolation does not  
75 bring in new information. Even worse, *additional* computa-  
76 tion is required in feature extraction.<sup>1</sup> On the other hand, in  
77 the case of scaling down, information is lost, although com-  
78 putation is saved in feature extraction. However, this saving  
79 in computation is at the cost of losing precious information.  
80 In the subspace morphing theory, since no normalization in  
81 scale is necessary, information is used “as is”, and no extra  
82 cost in additional computation is required.

83 *Identification capability:* Identification capability is  
84 defined as the number of object classes an algorithm can  
85 identify before it starts to be confused. Regardless of what  
86 features are used, to simplify the analysis, assume that each  
87 class has the same number of samples, say  $N_c$ . If the feature  
88 space is fixed, say  $R^n$ , the upper bound of the number of  
89 object classes these techniques are able to identify is  $C_{N_c}^n$ .  
90 In the subspace morphing theory, on the other hand, since  
91 the feature space  $R^n$  is not fixed, and it can be morphed  
92 to an arbitrary space dynamically in the process of identi-  
93 fication, this upper bound is open, and in general is much  
94 higher than the fixed upper bound, resulting in much larger

<sup>1</sup> The complexity of SVD is  $O(mn^2)$  for a matrix in  $m \times n$  [17].

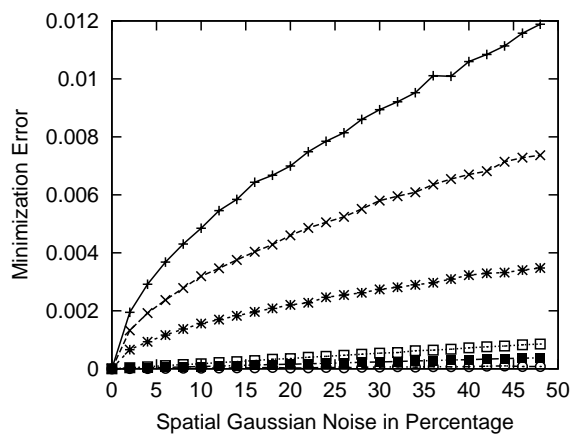


Fig. 1. Identification precision analysis based on noise corrupted intensity appearance. Simulated Gaussian noise is added into real query images at both spatial level (from 0% to 50%) and intensity level (5%, 10%, and 15%); the performances are computed by averaging over 1000 runs. The legends: +, x, \*—performances of the normalization based method after 15%, 10%, and 5% Gaussian noises are added into the intensity space, respectively; □, ●, ○—performances of SM after 15%, 10%, and 5% Gaussian noises are added into the intensity space, respectively.

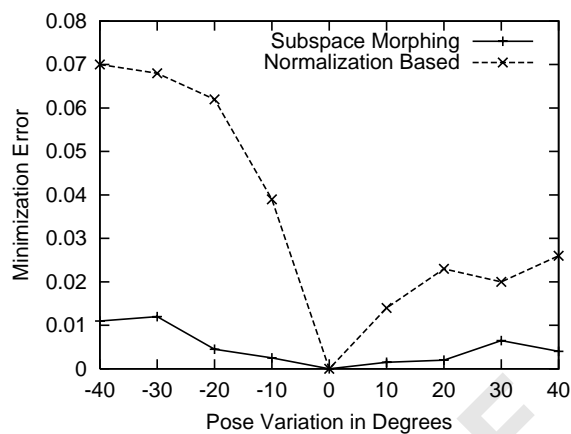


Fig. 2. Identification precision analysis based on pose change. Query images of nine different poses of the same object are input into both the normalization based method and SM for performance comparison.

1 identification capability than that of the normalization based  
2 techniques.

3 *Identification precision:* Identification precision refers to  
4 how much appearance change is allowed before misclas-  
5 sification happens. Clearly, if normalization is conducted  
6 to scale up an image, the identification precision stays the  
7 same, as data interpolation does not bring in new informa-  
8 tion. However, if normalization is to scale down an image,  
9 the identification precision degrades significantly. This is  
10 shown below in two types of appearance change: noise cor-  
11 ruption of original images and pose change of the same  
12 object. In both cases, images in the model base are both in half  
13 scale of the query images in row and column dimensions. In  
14 order to assure a fair comparison, a nearest neighbor based  
15 identification technique using SV features in the pre-defined  
16 model image space is implemented as a representative tech-  
17 nique of the conventional, normalization based approaches.

18 In order to show the appearance change due to the change  
19 of intensity values, Fig. 1 gives the performances of the  
20 conventional technique and SM, when simulated Gaussian  
21 noise is added into real query images in both spatial level  
22 and intensity level. The degradation is measured as the standard  
23 minimization error in both techniques, and the results are  
24 obtained after averaging 1000 runs. Clearly, SM exhibits  
25 more stable values under the same levels of Gaussian noise  
26 than the conventional technique does.

27 In order to show the appearance change due to the change  
28 of object pose, Fig. 2 gives the performances of the both  
29 techniques when a query image of an asymmetric industrial  
30 part deviates a model image of the same object in pose from

31  $-40^\circ$  to  $40^\circ$  with  $10^\circ$  in each step.<sup>2</sup> Again, the performance  
32 degradation of the conventional technique is obviously more  
33 significant than that of SM.

34 In summary, subspace morphing theory performs signifi-  
35 cantly better than the conventional, normalization based  
36 methods if images are provided in different dimensions. This  
37 conclusion is also supported by experimental evaluations  
38 based on real data shown in Section 4.

#### 39 4. Experiments

40 SM is implemented and has been used to identify differ-  
41 ent types of objects, including human faces, flowers, coffee  
42 mugs, space shuttles, toys, etc. Due to the appearance based  
43 nature, this method may be used to identify both 2D and 3D  
44 objects, both rigid and deformable objects. In this section,  
45 in order to show a performance comparison between this  
46 method and the conventional, normalization based methods,  
47 a human face database is used as a benchmark evaluation.

48 Note that the significance of the subspace morphing  
49 theory is that a better performance is achieved by taking  
50 advantage of the otherwise lost information in the nor-  
51 malization process when images are provided in different  
52 dimensions. In order to conduct a fair comparison through  
53 experiments, it would be ideal that a common, large scale  
54 testbed (such as FERET [20]) is available to test both the  
55 subspace morphing based technique and the conventional,  
56 normalization-based techniques. Regrettably, all the pub-  
57 licly existing face databases consist of normalized images,  
58 which nullifies the essential advantage of the subspace mor-  
59 phing theory. Based on this consideration, a special face  
60 database with images of different dimensions is constructed

<sup>2</sup> These images are obtained from University of Plymouth, UK.

1 for this purpose. This database consists of two disjoint parts.  
 2 A model base consists of 112 face images of 25 individuals,  
 3 and a testing base consists of 226 face images of the same  
 25 individuals. These images were contributed voluntarily

Table 1  
 Face identification results<sup>a</sup>

Methods	SM	NE	LE	NS	LS
NS	208	92	104	102	56
NF	18	134	122	124	170
IR	92%	41%	46%	45%	25%

<sup>a</sup>Number of images in the model base: 112. Number of images in the testing base: 226. Number of subjects in both bases: 25. NS: Number of testing images succeeded. NF: Number of testing images failed. IR: Identification rate.

5 by people, or obtained through the Internet. In the process of  
 6 data collection, special attention was paid to reflect signifi-  
 7 cant variations in scale and appearances. The distribution of  
 8 the number of images for each individual in the databases is  
 9 not uniform, depending on the availability of the collected  
 10 images from these individuals. Images in the model base are  
 11 kept as all frontal or near-frontal images, while images in  
 12 the testing base are of relatively large variations of appear-  
 13 ances and scales to test the scalability and robustness of the  
 14 evaluated algorithms.

15 In order to show a comparison study between **SM** and the  
 16 conventional techniques in this evaluation, four classic face  
 17 identification techniques are implemented: nearest neighbor  
 18 based eigenface (**NE**) [2], linear combination based eigen-  
 19 face (**LE**) [21], nearest neighbor based SVD (**NS**) [5], and  
 linear combination based SVD (**LS**) by combining [5] and

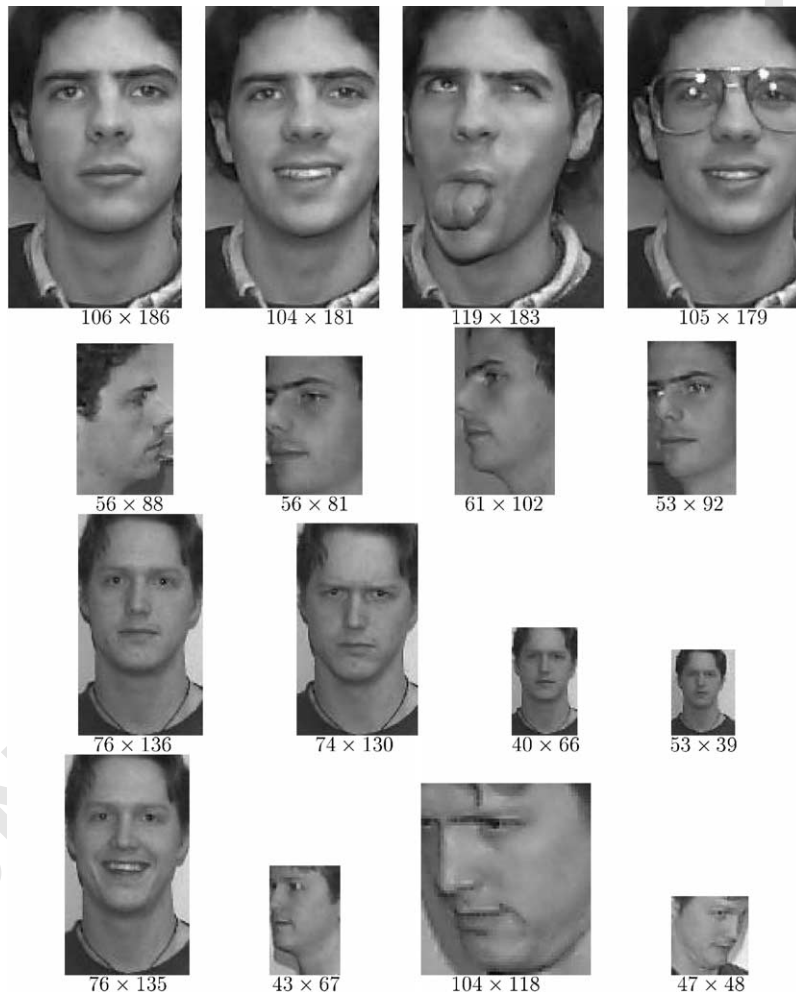


Fig. 3. Face examples in the experiments. The first and the third rows are the face images in the model database, and the second and the fourth rows are the images from the testing database; the dimensions of each images are indicated.

[21]. As all the conventional techniques require normalization in scale, all the 338 face images in the model and testing bases are scaled to a common dimension space of  $120 \times 80$  to generate a separate version of the database for the sake of running these four classic algorithms. Table 1 shows the identification results of **SM** and the four conventional techniques. The performance of **SM** is noticeably better than those of the four conventional identification techniques under this evaluation, even with large variations of appearances between the model images and the testing images. Fig. 3 shows several examples of the recognition results in which **SM** succeeded and all the four classic techniques failed. Note the large variations of the appearances between the model images and the testing images. Also note that all the images either from the model base or from the testing base are in different dimensions.

For those testing images that **SM** fails to identify, they were all those with significant pose variations with respect to the images in the model base. In fact, some of the examples shown in Fig. 3 are in this “borderline” category (i.e., with significant pose variations): some of them were correctly identified by **SM** such as those shown in Fig. 3 while others failed to be correctly identified by **SM**. In other words, those testing images with significant pose variations shown in Fig. 3 are “good” cases, and there were other similar images in the same category as bad cases. In particular, for those individuals with fewer images in the model base, their testing images with significant pose variations tended to fail to be correctly identified more often than those with more images in the model base, as the numbers of the collected images in the model base for each individuals are not equal due to the uneven availabilities in the collection of the images. However, noise of the images in the “borderline” category were correctly identified by any of the four classic algorithms. In general, this evaluation indicates that **SM** appears to have substantially larger tolerance towards scale and appearance variations (especially in “internal deformation” such as expressions, with or without glasses, different backgrounds, different hair styles, different clothes, etc.) than the classic techniques do. More specific conclusion in comparison of **SM** with the classic techniques requires further larger scale evaluations.

The evaluation result for **LS** is a bit counter-intuitive. It is believed that in general the nearest linear combination based techniques should perform better than the nearest neighbor based techniques [21]. This may require further investigation in the future.

## 5. Conclusion and future work

A new theory for appearance based object identification called subspace morphing is presented in this paper. It differs from most existing identification techniques in the sense that it does not require normalization in image scale, and consequently is scale invariant. Furthermore, since this the-

ory uses images “as is”, when images are provided in different dimensions, which is a typical and common situation in many applications, it is shown that the “extra” information in the difference of the image scales contributes to a better performance of this technique in object identification. This conclusion is supported by both theoretical analysis and experimental evaluation.

The future work will be focused on determining how far the subspace morphing theory can go in object identification with large variations between testing and model images, especially with significant variations in pose, either through theoretic analyses or through experimental evaluations. This will allow further extensive and more “accurate” applications of this theory in solving for general object identification problems.

## References

- [1] M. Turk, A. Pentland, Eigenfaces for recognition, *J. Cog. Neurosci.* 3 (1) (1991).
- [2] A. Pentland, B. Moghaddam, T. Starner, View-based and modular eigenspaces for face recognition, in: *Proceedings of the CVPR*, 1994.
- [3] B.J. Frey, A. Colmenarez, T.S. Huang, Mixtures of local linear subspaces for face recognition, in: *Proceedings of the CVPR*, IEEE Press, New York, 1998.
- [4] D.B. Graham, N.M. Allinson, Norm<sup>2</sup>-based face recognition, in: *Proceedings of the CVPR*, IEEE Computer Society Press, New York, 1999.
- [5] Z. Hong, Algebraic feature extraction of image for recognition, *Pattern Recognition* 24 (1991).
- [6] B.S. Manjunath, R. Chellappa, C.V.D. Malsburg, A feature based approach to face recognition, in: *Proceedings of the CVPR*, 1992.
- [7] R. Brunelli, T. Poggio, Face recognition: features versus templates, *IEEE Trans. Pattern Anal. Mach. Intell.* 15 (1993).
- [8] S. Lawrence, C.L. Giles, A.C. Tsoi, Convolutional neural networks for face recognition, in: *Proceedings of the CVPR*, 1996.
- [9] C. Liu, H. Wechsler, Evolutionary pursuit and its application to face recognition, *IEEE Trans. Pattern Anal. Mach. Intell.* 22 (6) (2000) 570–582.
- [10] C. Liu, H. Wechsler, Learning the face space—representation and recognition, in: *Proceedings of the ICPR, IAPR*, 2000.
- [11] G.J. Edwards, C.J. Taylor, T.F. Cootes, Improving identification performance by integrating evidence from sequences, in: *Proceedings of the CVPR*, IEEE Computer Society Press, New York, 1999.
- [12] N. Costen, T.F. Cootes, G.J. Edwards, C.J. Taylor, Simultaneous extraction of functional face subspaces, in: *Proceedings of the CVPR*, IEEE Computer Society Press, New York, 1999.
- [13] B. Moghaddam, A. Pentland, Probabilistic visual learning for object representation, *Trans. Pattern Anal. Mach. Intell.* (1997).
- [14] H. Schneiderman, T. Kanade, Probabilistic modeling of local appearance and spatial relationships for object recognition, in: *Proceedings of the CVPR*, IEEE, New York, 1998.

- 1 [15] A. Colmenarez, B. Frey, T.S. Huang, A probabilistic  
3 framework for embedded face and facial expression  
recognition, in: Proceedings of the CVPR, IEEE Computer  
Society Press, New York, 1999.
- 5 [16] I.J. Cox, J. Ghosn, P.N. Yianilos, Feature-based face  
7 recognition using mixture-distance, in: Proceedings of the  
CVPR, 1996.
- 9 [17] G.H. Golub, C.F.V. Loan, Matrix Computations, 2nd Edition,  
The Johns Hopkins University Press, Baltimore, MD, 1989.
- [18] S. Lang, Linear Algebra, Springer, Berlin, 1987.
- [19] S.E. Umbaugh, Computer Vision and Image Processing, 11  
A Practical Approach Using CVIP tools, Prentice-Hall,  
Englewood Cliffs, NJ, 1998. 13
- [20] P. Phillips, H. Moon, P. Rauss, S. Rizvi, The feret evaluation  
methodology for face-recognition algorithms, in: Proceedings  
of the CVPR, 1997. 15
- [21] S.Z. Li, Face recognition based on nearest linear combinations,  
in: Proceedings of the CVPR, IEEE, New York, 1998. 17

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