Modeling and Analysis of Two-Flow Interactions in Wireless Networks

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Abstract

Interference plays a complex and often defining role on the overall performance of wireless networks, especially in multi-hop scenarios. Understanding this role is critical for understanding these networks, and in turn for developing effective protocols for them. In the presence of interference, Carrier Sense Multiple Access MAC protocols are known to suffer from the hidden terminal and exposed terminal problems, which can cause poor performance and unfairness. Recent work has shown that depending on the relative location of interfering sources and destinations, several modes of interference exhibiting different behavior, occur. In this paper, we first relax the assumption that the interference range is equal to the reception range. This gives rise to a large number of interference configurations; we develop closed form expressions for their frequency of occurrence. As a result, we discover that the frequency of occurrence of the major modes of interference change significantly from those obtained without relaxing the interference range assumption. More importantly, we show that two previously unknown modes of interactions arise, whose performance differs significantly from the known modes. We develop models for estimating the throughput for the different categories of interaction, and validate them against simulation results. We believe that this analysis represents a further step into the understanding and characterization of the impact of interference from first principles.

1 Introduction

In Multi-Hop Wireless Networks (MHWNs) that use Carrier Sense Multiple Access (CSMA) MAC protocols, different forms of hidden terminal and exposed terminal problems arise [1], which can lead to poor link quality, and short term or long term unfairness. Complex interactions occur between interfering links based on the relative location of the senders and receivers (more accurately the state of the channel between them). These interactions play an important role in determining the link quality, and can give rise to sustained or short term unfairness. Understanding these interactions is critical for understanding and characterizing behavior in MHWNs and for designing effective protocols for them.

Recent work has analyzed and classified the different behaviors that arise between two interfering links that use the IEEE 802.11 protocol [7,13]. Understanding and characterizing interactions at this level is a promising first step towards an understanding of the effect of interference from first principles.

This paper makes several contributions for improving the analysis of two-flow interference, using more realistic assumptions, identifying additional types of interactions, and analytically modelling their behavior. Specifically, we make the following contributions:

1. *Generalizing the analysis by allowing Interference/Carrier Sense range being different from reception range (Section 4).* Relaxing this assumption results in a large number of individual scenarios[1]. [7,13] show 16 different interactions while our study produce 53 different interactions which can be grouped based on the type of interaction into 5 different classes.

2. *Geometric analysis, leading to closed form expressions, for the probability of occurrence of the scenarios (Section 5).* In contrast to the existing geometric models [7], we use a new simpler approach that allows direct evaluation of the probability of the grouped cases (avoiding the need to model each of the individual 53 scenarios). We also remove the need to condition the analysis on the distance between the sender and receiver, allowing derivation of the general probability of the scenarios. The geometric models validate very well to a Monte Carlo characterization of the probability.

3. *Analytical models characterizing the performance of the different classes of interactions (Section 6).* The models are validated using simulation. This includes models for the two new identified classes of interaction.

4. *Preliminary analysis with extensions of the model to analyze self-interference in multi-hop chains (Section 7).* While we intend to provide a more detailed treatment of these important extensions in our future work, we provide some initial results with them. We show that in chains, destructive interactions occur with different probabilities than the general two flow cases. We also show that they have important implications on the chain performance, motivating work in routing protocols to discover routes with deeper understanding of the impact of interference.

We believe that these contributions collectively enhance the understanding of causes and impact of interference. However, several important steps remains towards a generalization of this understanding, including the use of a more realistic channel model and experimental validation of the results. We present our conclusions and thoughts about future work in Section 8.

2 Related work

Carrier sense multiple access (CSMA) [9] based protocols such as IEEE 802.11 are heavily used for medium access in wireless networks. CSMA works by attempting to prevent sources from transmitting concurrently by having each source sense the channel before transmission. In wireless networks, the state of the channel at the receiver is what determines whether a reception occurs successfully. As a result, carrier sense at the sender does not accurately reflect the state at the receiver. More precisely, if the receiver channel is busy but the sender channel appear idle, a collision occurs – *the hidden terminal problem*. Conversely, if the receiver channel is available but the sender channel appears busy, the transmission is unecessarily deferred – *the exposed terminal problem*. These problems are known to significantly degrade the performance of CSMA in wireless settings [9].

A number of protocols have been proposed to attempt to reduce the effect of the hidden terminal and exposed terminal problems. Wu and Li propose using a busy tone channel that is used by the receiver when it is receiving a packet [16]. Other sources sense this channel allowing carrier sense based on the receiver position. Karn proposed the MACA protocol (Multiple Access Collision Avoidance), a predecessor

[1]Based on the 3 possible states (out of range, in reception range, or in interference range) of the four secondary links (S1-S2, D1-D2, S1-D2, and S2-D1), there are a total of 3^4 scenarios. After eliminating the redundant states which differ only by relabelling the connections, a total of 53 unique scenarios result.

to the current IEEE 802.11 protocol [8]. MACA uses short request-to-send (RTS, sent by the sender before transmission) and clear-to-send (CTS, sent by the receiver if RTS is received correctly and the channel is available) to attempt to reduce collisions. Potential interferers that receive the RTS or the CTS packet do not transmit for the duration of the packet. Bhargavan et al's MACAW protocol improves MACA by adding acknowledgement and retransmission [1].

Despite these advances in protocol design, the hidden terminal problem continues to plague CSMA MAC protocols. In fact, depending on the relative location of the senders and receivers, and other factors such as the MAC protocol, and the interference model, a number of interaction modalities with distinct behavior occur. Bharghavan et al identify and discuss several of these cases and propose modifications to the MACAW protocol to address them individually [1] in a network where the interference range is equal to the reception range.

Our work is most related to the following two efforts that attempt to methodically characterize and analyze the performance of the different modes of interactions that occur between two interfering links. Rogers and Abu-Ghazaleh [13] conduct a simulation study of all the possible configurations of two interferening links under saturation traffic and with a fixed interference range which is significantly larger than the reception range. They enumerate the possible modes of interaction, and discover a number of cases with destructive interactions both with RTS/CTS and without RTS/CTS.

Most relevant to our work, Garetto et al enumerated the types of interactions that occur under assumptions of transmission range equal to interference range, and developed geometric models for analyzing their expected frequency [7]. They also presented analysis the performance using simulation. They grouped the scenarios into 3 general classes based on their behavior. We generalize their analysis in a number of important ways, discovering two new modes of interactions, which leads to a more realistic characterization of the impact of interfernece on CSMA protocols.

Models for computing throughput in CSMA networks were studied intitially by Boorstyn et al. [3] and Tobagi et al. [14]. Advanced models for calculating the throughput in IEEE 802.11 based networks have been proposed [4, 6, 10, 15]. Even though these works account for the effect of interactions, they do so for specific networks and using iterative methods. In contrast, the focus of the paper is to classify and analyze all the possible interactions between two contending links and to model the resulting behavior constructively from first principles, based on the interactions that cause them.

3 Background and Existing Models

Garetto et al [7] categorize the two-flow interactions using a boolean physical model where the transmission radius is equal to the interference radius. In this scenario, the nodes for each secondary link can be either in range or out of range, leading to 2^4 different scenarios corresponding to the different combinations of states that each of the four links can be in. The 16 scenarios can be reduced to 13 by eliminating the dual scenarios (scenarios that are identical other than relabelling the connections). They compute the occurrence probability of each of the scenarios conditioned on a fixed distance between the primary senders and receivers. More interestingly, they recognize that the individual scenarios can be grouped into three basic categories described below.

Sender-Connected (SC): This category includes all scenarios where the two senders are within interference range. In SC scenarios, a sender will not start a transmission when the other sender is active due to CSMA and no collisions other than those when the two senders start transmission at the same time will occur (such collisions are of low probability due to the randomization of the backoff period incorporated by IEEE 802.11). Figure 1(a) shows an example SC scenario.

Asymmetric Incomplete State(AIS): In the remaining scenarios the senders are not connected (Incomplete State). A distinguishing attribute is whether the state of the S_1, D_2 and S_2, D_1 states are identical (Symmetric) or different (Asymmetric). In Asymmetric Incomplete State, only one of the senders can interfere with the other destination. Thus, only one of the flows experiences packet collisions. Figure 1(c) shows a sample scenario for AIS where the flow (S_2, D_2) experiences a packet collision from S_1. An *incomplete state* is created since the the source of the weaker link (S_2) does not have complete information about the channel at its destination (D_2). Having a complete information could have prevented the packet collision for (S_2, D_2).
Symmetric Incomplete State (SIS): Under this category, the senders are not connected. However, either both the senders can interfere with the other destination, or they cannot. In these scenarios, short term unfairness may arise, but no bias exists to lead to long term unfairness between the connections. Figure 1(e) shows a sample scenario for SIS where the flow (S_1, D_1) and (S_2, D_2) interfere with each other.

4 Categorizing Two-flow Interactions

This section presents the categories of interaction that arise when we relax the assumption of the interference/carrier sense range being equal to the communication range. Specifically, we assume that a sending node S_1 causes a collision at a receiving node D_1 if they are within interference range with each other. As a result, the possible states of the four secondary flows (S_1S_2, S_1D_2, D_1S_2 and D_1D_2) now become: (1) in communication range; (2) in interference range, but not in communication range; (3) out of range.

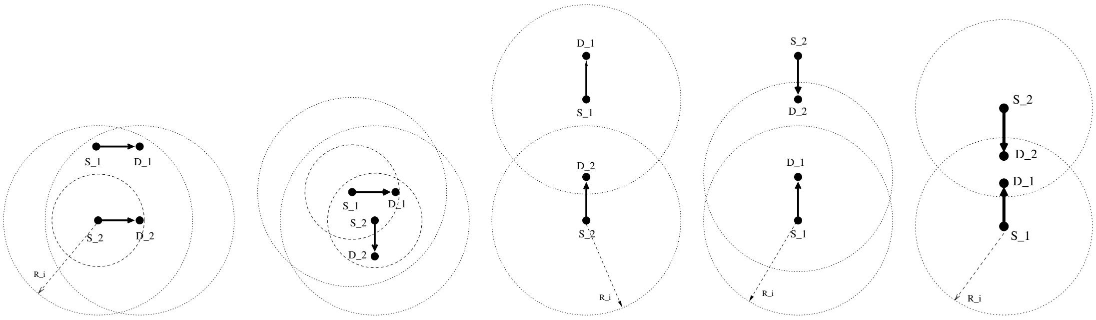
Each of the four cross links can be in one of the above 3 states relative to each other for a total of 3^4, or 81 enumerable scenarios. After removing the dual scenarios which are identical other than relabelling of the connections, a total of 53 distinct scenarios occur. For example the scenario where S_1D_2 in interference range with all other links out of range is the dual of the scenario where S_2D_1 is in interference range with all other links out of range.

Luckily, the scenarios share important characteristics that allows us to classify them into a small number of categories (5 in this case). We call scenarios where the two senders are within interference range (including those within communication range) *Sender Connected*. In such scenarios, the two senders arbitrate the channel successfully and fairly. The term *Symmetric* is used when each of flow interacts with the other flow symmetrically; in other words, the state of the S_1D_2 link is identical to the S_2D_1 link. Like before, we use *Incomplete State* to mean that the senders are not connected.

Due to the possibility of links being in interference but not communication range, the categories of scenarios exhibiting different interference behavior grow from three to five. In the following we discuss the five categories in more detail.

1. **Senders Connected Symmetric Interference (SCSI):** SCSI represents sender connected scenarios where there is symmetric interference between opposite source and destination. For example, if link S_1D_2 is in interference range then D_1S_2 is also in interference range. Figure 1(a) shows a sample SCSI scenario. Flows in this group share the medium fairly due to symmetry.

2. **Senders Connected Asymmetric Interference (SCAI):** this subset of scenarios represent the first new category of interaction that we identify. In SCAI: (1) the senders are within communication range of each other; (2) One sender and the opposite receiver (belonging to the other flow) are in interference range (say, $S_1D_2 \leq R_i$ in Figure 1(b)); and (3) The other sender and receiver are not in interference range of eachother.



(a) SCSI (b) SCAI (c) AIS (d) IDIS (e) SIS

Figure 1: Sample scenarios in each category

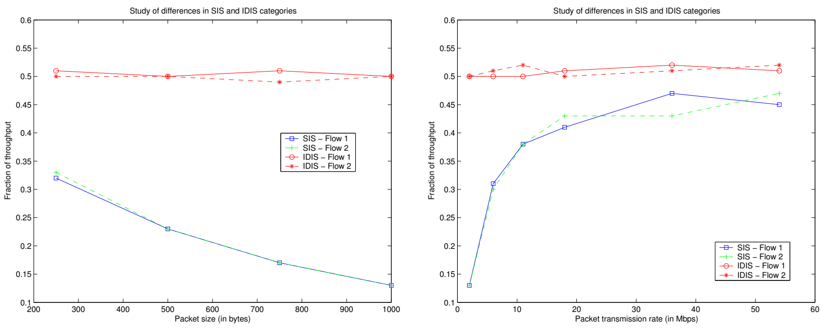
Figure 1(b) shows a sample scenario in SCAI where S_1 and D_2 are in interference range, but not in communication range. Under the IEEE 802.11 protocol, the source S_1 can sense the channel busy when D_2 sends an ACK packet to S_2, but cannot decode the packet. It perceives such busy signal as an ongoing transmission. In order to avoid a possible collision, S_1 will wait for the channel to be idle for EIFS period (a significantly larger period than the standard DIFS inter-frame separation to ensure completion) before it transmits a packet.

S_2 on the other hand receives the ACK properly from D_1 and will only wait the DIFS duration (which is $\ll EIFS$) before decrementing its backoff. Since its backoff timer is much shorter than EIFS timer, S_2 gets an unfair advantage in channel contention, wins the channel again and the cycle continues. This causes severe unfairness in two links and link S_1D_1 starves.

3. **Asymmetric Incomplete State (AIS):** This scenario is identical to the AIS category in the original classication. Briefly, in these scenarios, one of the flows (say, (S_1, D_1)) will cause a packet collisions to the other flow but not vice versa. Thus, (1) the senders are out of range and can transmit simultaneously; and (2) One source and the opposite receiver are in interference range of each other and (3) The second source and its opposite receiver are out of range of eachother. Figure 1(c) shows a sample AIS scenario. Many of the packets sent to D_2 are lost because of interference from S_1, while D_1 receives all packets from S_1 successfully.

4. **Interfering Destinations Incomplete State (IDIS):** This is the second newly identified category of interactions which is a subset of the originally classified SIS cases. This group includes scenarios where all the secondary links are out of range except the two destinations. Figure 1(d) shows one such scenario. Since both the sources are out of range (not sender connected), they transmit packets simultaneously. The destination that receives its packet sends an ACK, thus causing a collision for the ongoing packet transmission at the other destination. This causes short term unfairness for each link. IDIS is a *Sender Unconnected*, *Symmetric* and *Incomplete state* scenario that experiences drops due to ACK packets.

5. **Symmetric Incomplete State (SIS):** The senders are out of range and both sets of opposite source and destination are within communication or interference range. Figure 1(e) shows a scenario with SIS. This problem causes the overall throughput of the links to decrease substantially without affecting the fairness issue. Since the two senders are out of range, they will transmit simultaneously. Since each



(a) Throughput vs. Packet Size (b) Throughput vs. Transmission rate

Figure 2: Comparison of SIS and IDIS

destination can be interfered by the opposite source, there is a packet drop at both the destination. This will cause significant low throughputs at for both links.

Scenarios belonging to IDIS and SIS groups are categorized as SIS group in Garetto et al [7]. However, owing to difference in the interactions and throughput of the links, we classify them into separate groups. In SIS, since the two sources will backoff at different levels independent of each other, the only opportunity for a source to successfully send a packet is when the source starts and ends the transmission within the backoff period of the other source. This should allow for better throughputs for shorter packet (smaller in size or higher rate). This is different from IDIS where the links will have short term unfairness regardless of packet size. Figure 2 compares the throughput of SIS and IDIS. As we increase the packet size the throughput of SIS group decreases while that of IDIS group remains unchanged (Figure 2(a)). As the transmission rate is increased, the amount of time required to send the packet reduces. Hence throughput of SIS increases (Figure 2(a)).
The above interactions do not consider the use of RTS/CTS. With RTS/CTS, additional modes of interaction arise since RTS/CTS can be received by nodes in communication range, but not those in interference range. However, in practice most IEEE 802.11 networks disable the RTS/CTS option since it cannot prevent many interferes that are outside of communication range. Thus, we decided not to pursue the additional interactions that arise in that mode.

5 Determining Scenario Probability

In this section, geometric models are developed to predict the probability of occurrence of the scenario groups identified in the previous section. Due to the increased number of cases, and the increased complexity of each case due to the addition of a separate interference range, we develop a completely new and simpler approach than the one used by Garetto et al [7]. The problem is one of estimating different regions of intersection of the circles forming the communication and interference ranges of the different nodes, which correspond to the interactions scenarios. The Garetto approach would require a complex case by case treatment of the 53 scenarios; our model allows us to capture the probability of the 5 categories directly. In addition, we formulate the more general probability of the cases given only that the two connections interact whereas the expressions developed by Garetto et al. are conditioned on a given distance between the sender and receiver.

5.1 General Approach and Preliminaries

We define the interference range and communication range as r_i and r_c respectively. The radius of the whole network is represented by r_s. From the structure of the scenario, since D_1 is the destination of S_1 for one flow these two nodes are always within r_c of each other, and similarly D_2 is always within r_c of S_2. We assume a two disc binary model where a node inside the communication range will receive a message without any errors and a node transmitting from interference range will cause all packets to be dropped at the receiver. We realize that using Signal to Noise ratios SINR is a more accurate measure for determining packet reception and is a topic of our future work. We also assume a uniform distribution of nodes in the network. We use the following terminology: $C(X)$ refers to the area of communication range of X (circle of radius r_c around X) and $T(X)$ refers to the interference range of X (circle of radius r_i around X). One thing to note is that we have picked r_s to be $2r_c + r_i$, this is the minimum network size to capture all possible scenarios. Increasing the size from here will only increase the percentage of cases with no interactions and hence our evaluation is independent of network size.

In general, the derivation requires computing the area of intersection of two or three circles of different radii. While the area of intersection of two circles is well known, computing the area of intersection of three circles is a surprisingly difficult problem. Fortunately, Fewell recently developed expressions for the intersection of three circles, which we apply in our models [5].

For the four secondary channels (S_1 to S_2, D_1 to D_2, S_1 to D_2 and S_2 to D_1), the general approach requires computing the probability of the presence (or absence) of a node within r_c or r_i from other nodes concurrently as appropriate for the case being modeled. For example, for the SCSI category where S_1 and S_2 are in interference range but the other three secondary links are out of range with each other, this requires computing the area of intersection of $T(S_1)$ and $T(S_2)$ that is also outside any of the intersections of $T(S_1)$ and $T(D_2)$, and $T(S_2)$ and $T(D_1)$. This intersection must be computed over all possible distances between S_1 and D_1 and S_2 and D_2.

5.2 Example: IDIS Probability Derivation

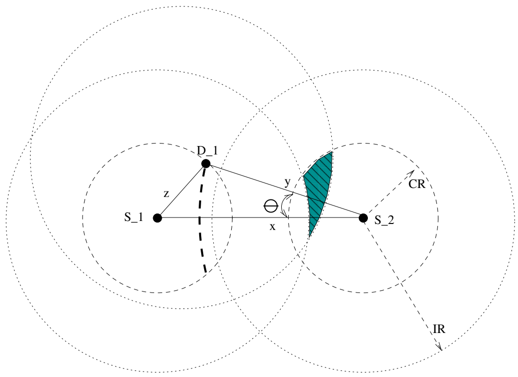


Figure 3: IDIS Group Example

For the IDIS group, only the destinations are within range of each other (interference or communication) while all other nodes are out of range. We first compute the probability of the two senders being out of range of each other. Then we find the probability that one destination is out of range of the opposite sender. Then we find the probability that the second destination is out of range of its opposite sender as well as the probability that two destination are in range.

To compute the probability of IDIS we have to calculate *(a)* The probability that the two sources are out of range of each other. *(b)* The probability that both destinations are out of range of the opposite sources and *(c)* Given the constraints of (b), the two destinations are within range of each other. To compute (a), the probability that S_2 is at a distance x from S_1 in a network of radius r_s is given by $\frac{2x}{r_s^2}$, integrating this equation from r_i to r_s will give us the probability p_1 of S_2 being out of range of S_1. More precisely,

$$p_1 = \int_{r_i}^{r_s} \frac{2x}{r_s^2}\, dx \tag{1}$$

We divide (b) in two parts, probability that D_1 is out of range of S_2 and probability that D_2 is out of range of S_1. First we find the probability of the first part and then we combine the second part with (c). Lets assume that D_1 is at a distance y from S_2, we find the probability that D_1 is on an arc at a radius of y from S_2 which is given by

$$\frac{y\; dy\; d\theta}{C(S_1)} \tag{2}$$

where dy is the width of the arc and θ is the angle $\angle D_1\ S_2\ S_1$ as shown in the figure. Because of symmetry we will only consider the values where θ is positive and then we multiply by 2 to get the lower half. Since D_1 has to be in communication range of S_1, the limits of θ are from 0 to $\theta_{\max}$ which is computed as follows.

$$\theta_{\max} = \arccos \frac{x^2 + y^2 - r_c^2}{2xy} \tag{3}$$

Since we are interested in D_1 being out of range of S_2 the distance y has a lower limit of r_i. Its possible for larger values of x, arc of radius y around S_2 will not intersect circle of radius r_c around S_1, to take care of this case we take the lower limit of y to me the maximum of r_i and $x - r_c$. The maximum value that y can take is $x + r_c$. Integrating eq 2 from 0 to $\theta_{\max}$ with respect to θ and from r_i to $x + rc$ with respect to y will give us the probability of D_1 being out of range of S_2

$$p_2 = \int_{max(r_i, x-r_c)}^{x+r_c} \int_0^{\theta_{\max}} \frac{y d\theta dy}{C(S_1)} \tag{4}$$

To find the probability of D_1 and D_2 being in range we find the area of intersection $(A(S_2 \cap D_1))$ of the circle with radius r_i around D_1 and the area of the circle with radius r_c centered at S_2. Dividing this area by $C(S_2)$ will give us the probability that D_1 and D_2 are within range. This probability will include those cases where S_1 and D_2 are within range. To remove these cases we subtract from $(A(S_2 \cap D_1))$ the area of intersection of circles of radii r_i around S_1, r_i around D_1, and r_c around S_2.

$$p_3 = \frac{(C(S_2) \cap T(D_1)) - C(S_2) \cap T(S_1) \cap T(D_1)}{C(S_2)} \tag{5}$$

The area of intersection of three circles $Eq16$ in [5] requires that the distances between the center of the circles and their radii are known. The distance between S_1 and D_1 is the only unknown in our case which can be calculated by using x and y and the angle θ between this two line by using the law of cosines

$$z^2 = x^2 + y^2 - 2xy \cos\theta \tag{6}$$

Combining equations 1, 4, and 5 we get the overall probability of IDIS group by:

$$P(IDIS) = \int_{r_i}^{r_s} \int_{max(r_i, x-r_c)}^{x+r_c} \int_0^{\theta_{\max}} p_3 \frac{2xy}{r_s^2 C(S_1)} d\theta dy dx \tag{7}$$

5.3 SCSI Group

For SCSI, the two senders have to be within range of each other (interference or communication). The two-flows will belong to this group if

- The two senders are in interference range or
- the two senders are in communication range and
  - each sender is in interference range of the other destination or
  - each sender is in communication range or out of range of the other destination.

To find the probability of the occurrence of this scenario first we calculate the probability of two sources being in interference range. We can get this probability by integrating $\frac{2x}{r_s}$ from r_c to r_i as given in the following equation.

$$p_{S_1 S_2 int} = \int_{r_c}^{r_i} \frac{2x}{r_s^2} dx \tag{8}$$

integrating the same equation from 0 to r_c will give us the probability that the two sources are in communication range.

$$p_{S_1 S_2 com} = \int_0^{r_c} \frac{2x}{r_s^2} dx \tag{9}$$

If the two senders are in communication range, the probability that D_2 is in interference range of S_1 is given by

$$p_1 = \frac{T(S_1) \cap C(S_2)}{C(S_2)} \tag{10}$$

Hence 1-p_1 is the probability that D_2 is not in interference range of S_1 (in communication range or out of range). Probability that both receiver are in interference range of opposite sender is $p_1 p_1$ and the probability that both receivers are not in interference range of opposite sender is $(1 - p_1)(1 - p_1)$. Hence if the two sources are within communication range of eachother we can find the probability that SCSI occurs as p_1^2 plus the probability $1 - p_1^2$ times the probability that two sources are in communication range. The total probability of SCSI is given by:

$$p_{SCSI} = \int_{r_c}^{r_i} \frac{2x}{r_s^2} dx + \int_0^{r_c} (p_1)^2 (1 - p_1)^2 \frac{2x}{r_s} dx \tag{11}$$

5.4 SCAI Group

SCAI group represents scenarios where the two senders are in communication range and one of the destinations is in interference range of the opposite sender. The other destination is either in communication range or out of range . To calculate the probability of this group we note that integrating eq 9 gives us the probability that two senders are in communication range. The probability that one destination is within interference range of the opposite sender is calculated as the area of intersection of interference range of the opposite sender and the area of communication range of the sender divided by the communication range of the sender. Mathematically

$$p_1 = \frac{(T(S_1) \cap C(S_2)) - (C(S_1) \cap C(S_2))}{C(S_2)} \tag{12}$$

Also the probability that the other destination is either in communication range or out of range of the opposite sender is given by p_2 and p_3 respectively,

$$p_2 = \frac{C(S_2) \cap C(S_1)}{C(S_1)} \tag{13}$$

$$p_3 = \frac{C(S_1) - (T(S_2) \cap C(S_1))}{C(S_1)} \tag{14}$$

Since there is a symmetric possibility of D_2 being in interference range of S_1, we need to multiple the total probability by 2.

$$p_{SCAI} = \int_0^{r_c} 2p_1(p_2 + p_3)\frac{2x}{r_s^2}dx \tag{15}$$

5.5 AIS Group

In AIS, the senders are out of range and one source and the opposite destination are with in range (interference or communication range) while the other source and its opposite destination are out of range of each other.

Eq 1 gives us the probability that the two senders are out of range.

Following equation gives the probability that a destination is out of range of the opposite source

$$p_1 = \frac{C(S_1) - (T(S_2) \cap C(S_1))}{C(S_1)} \tag{16}$$

while the probability that a destination is within range of opposite source is given by.

$$p_2 = \frac{(T(S_1) \cap C(S_2)}{C(S_2)} \tag{17}$$

Multiplying these probabilities and adding a factor of 2 (since we can have the reciprocal case also) we get the total probability of AIS group.

$$p_{AIS} = \int_{r_i}^{r_s} 2p_1p_2\frac{2x}{r_s^2}dx \tag{18}$$

5.6 SIS Group

In SIS group, the senders are out of range and the both destination are with in range (interference or communication) of opposite senders. For the two sender to be out of range at a distance of x from eachother we can use eq 1. The probability that destination $D1$ is at a distance y from $S1$ on an arc intersected by circle of radius ri around $S2$ is given by dividing the length of the arc by the area $C(S1)$. This would make sure that $D1$ is in interference range of $S2$.

$$p_1 = \frac{2ycos^{-1}(y^2 + x^2 - ri^2)}{2xyC(S1)} \tag{19}$$

Similarly we calculate the probability of $D2$ being in interference range of $S1$ while at a distance of z from $S2$ as

$$p_2 = \frac{2zcos^{-1}(z^2 + x^2 - ri^2)}{2xzC(S1)} \tag{20}$$

By integrating these two probabilities over y and z and multiplying with 1 we get total probability of SIS group as

$$p_{SIS} = \int_{r_i}^{r_s} \int_0^{r_c} \int_0^{r_c} \frac{2x}{r_s^2} p_1 p_2 dx \tag{21}$$

5.7 Validation of the Geometric Models

We validate the geometric models that were developed for the five categories against exhaustive enumeration of the cases. Specifically, S_1 is placed at a fixed location. D_1 is moved around S_1 in the entire area of a circular disc with radius equal to the communications range. For every placement of S_1 and D_1, we move S_2 around S_1 in an area of circular disc of radius $(r_i + 2r_c)$. For each location of S_2, we place D_2 in the circular area of radius r_c around S_2. Note that the above procedure, encompasses all the possible legal locations of S_2 and D_2 such that at least one of them interacts with one of S_1 or D_1. However, it also results in some cases with no interaction which are not of interest to us. We remove those cases from the total count of legal cases to compute the probability. For each of the scenarios we evaluate the interaction between each link to produce the total number of times each scenario will occur. An alternative approach is to try a true Monte Carlo solution where the nodes are dropped randomly and the interactions evaluated. Table 1 shows all possible distinct interactions Scenarios are divided into groups as follows

Figure 4 plots the occurrence probability of the different groups. We make the following observations:

- The closed form analysis matches closely the results obtained via exhaustive enumeration at all interference ranges.
- If the interference range is set same as the communication range, the probability of SCSI increases while the probability of SCAI decreases to 0. As we increase the interference range, while keeping the senders connected, a higher percentage of the area of interference of one source overlaps the area of communication of the other source. This allows for a higher percentage of asymmetrically connected destinations, hence increases SCAI and decreases SCSI.

Scenario	Communication Range	Interference Range
1	S1-S2 S1-D2 D1-S2	D1-D2
2	S1-S2 S1-D2 D1-S2	D1-D2
3	S1-S2 S1-D2 D1-S2	
4	S1-S2 S1-D2 D1-D2	
5	S1-S2 S1-D2	D1-D2
6	S1-S2 S1-D2	
7	S1-S2 D1-D2	
8	S1-S2 D1-D2	S1-D2 D1-S2
9	S1-S2	D1-D2
10	S1-S2	S1-D2 D1-S2 D1-D2
11	S1-S2	S1-D2 D1-S2
12	S1-S2	
13	S1-D2 D1-S2 D1-D2	S1-S2
14	S1-D2 D1-S2	S1-S2
15	S1-D2 D1-S2	S1-S2 D1-D2
16	S1-D2 D1-D2	S1-S2 D1-S2
17	S1-D2 D1-D2	S1-S2
18	S1-D2	S1-S2 D1-S2
19	S1-D2	S1-S2 D1-S2 D1-D2
20	S1-D2	S1-S2 D1-D2
21	S1-D2	S1-S2
22	D1-D2	S1-S2 S1-D2
23	D1-D2	S1-S2
24	D1-D2	S1-S2 S1-D2 D1-S2
25		S1-S2 S1-D2 D1-S2 D1-D2
26		S1-S2 S1-D2 D1-S2
27		S1-S2 S1-D2 D1-D2
28		S1-S2 S1-D2
29		S1-S2
30		S1-S2 D1-D2
31	S1-S2 S1-D2 D1-D2	D1-S2
32	S1-S2 S1-D2	D1-S2 D1-D2
33	S1-S2 S1-D2	D1-S2
34	S1-S2 D1-D2	S1-D2
35	S1-S2	S1-D2 D1-D2
36	S1-S2	S1-D2
37	S1-D2 D1-D2	
38	S1-D2	D1-D2
39	S1-D2	
40	D1-D2	S1-D2
41		S1-D2 D1-D2
42		S1-D2
43	D1-D2	
44		D1-D2
45	S1-D2 D1-S2 D1-D2	
46	S1-D2 D1-S2	
47	S1-D2 D1-S2	D1-D2
48	S1-D2 D1-D2	D1-S2
49	S1-D2	D1-S2 D1-D2
50	S1-D2	D1-S2
51	D1-D2	S1-D2 D1-S2
52		S1-D2 D1-S2 D1-D2
53		S1-D2 D1-S2

Table 1: Table of all possible distinct two-flow scenarios

Scenarios	Group
1-30	SCSI
31-36	SCAI
37-42	AIS
43-44	IDIS
45-53	SIS

Table 2: Table of scenarios grouped into their respective classes

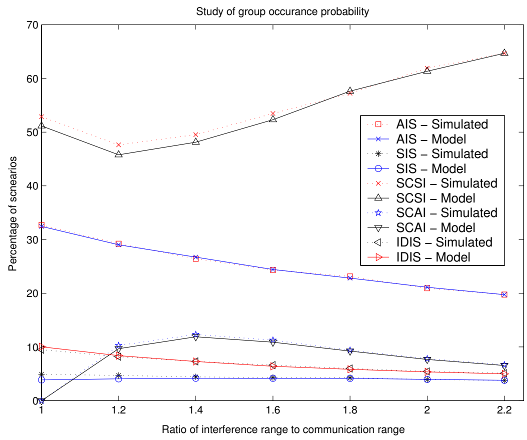


Figure 4: Occurrence Probability of the Groups

- As interference range grows further the percentage of interference range of a source overlapping the communication range of the other source decreases and hence the probability of SCAI decreases. The same fact also contributes to a decrease in the probability of IDIS.
- The AIS group decreases as interference range increases because the probability of a destination being in the communication range of the opposite source stays the same while the probability that it is in the interference range increases. Hence the probability that both destination are in interference range of the opposite source increases contributing more towards the SCSI group and taking away from AIS group.
- The SIS group probability remains relatively constant with changing interference range.

6 Throughput Estimation Model

In this section, we propose a model for the computation of throughput for the proposed categories. We derive the throughput model under a homogeneous network where the all the nodes have the same MAC level parameters. The channel capacity is denoted by C. The minimum and maximum backoff window is represented by $\mathrm{CW}_{\min}$ and $\mathrm{CW}_{\max}$, respectively. The packet loss probability given that the link transmitted a packet (conditional collision probability [2]) is represented by p. The probability that a source node starts transmission during an idle slot is denoted by τ. Bianchi [2] derived the expression for τ under Binary

Exponential Backoff (BEB) as a function of p (Equation 22).

$$\tau = \frac{2q(1-p^{m+1})}{q(1-p^{m+1}) + \mathrm{CW}_{\min}[1-p-p(2p)^{m'}(1+p^{m-m'}q)]} \tag{22}$$

where $q = 1 - 2p$, m is maximum number of retries and m' is the number of stages to reach $\mathrm{CW}_{\max}$ ($m' \leq m$). The p and τ for the link i is denoted by p_i and τ_i respectively. Bianchi's model accounts for the probability of transmission in a given slot based on the binary exponential backoff model.

We make the following assumptions: (1) The traffic on both the links is saturated. Under less than saturated assumptions, the interactions will play a less important role. It should be possible to extend the model to account for different packet assumptions; and (2) The nodes use the basic mode of IEEE 802.11 (without RTS/CTS), which is becoming the default mode in the network cards due to its superior performance in a majority of the scenarios. Extension of the model by relaxing the above assumptions is an area of future work.

For the SCSI, where the links have a fair-share of the channel without the hidden terminals, the throughput can be directly estimated using techniques similar to Single-hop wireless network (for example, Bianchi's model [2]). We briefly show the derivation of the model for the four other categories.

6.1 SCAI formulation

Under SCAI category, the sources are within interference range of each other and hence the transmission from the sources will not overlap. However, the EIFS effect causes one of the links (which we refer to as the 'weaker link') to wait for longer times before decrementing the backoff, thus causing throughput degradation.

Let τ_1 (τ_2) be the probability that the source of the weaker (stronger) link transmits the data packet, conditioned on the channel being idle. Since the links share a common channel, the probability of winning the channel for transmission by the weaker and the stronger link are in the ratio $\tau_1 : \tau_2$. Both the links suffer no hidden terminals ($p = 0$ for both links). Hence, the throughput of the link i is given by the Equation 23. l_i and o_i denotes the data payload size and the overhead size, respectively.

$$T_i = C\frac{\tau_i}{\tau_1 + \tau_2} \cdot \frac{l_i}{l_i + o_i} \tag{23}$$

The second part of Equation 23 denotes the fraction of the transmission that is used to send the payload. The stronger link always transmits with the same probability *when the channel is idle*. Hence, τ_2 can be calculated by equation 22. The only variable to be computed is τ_1 to determine the throughput of both the links.

Since we are interested in calculating the *transmission probability conditioned on the channel being idle*, we ignore the time during which the channel is busy. An idle slot can be in one of the backoff/EIFS states (a countable state space). And, the weak link will transmit when the backoff counter is zero (a subset of the state space). Hence, we use a discrete time Markov chain to calculate the probability of transmission at an idle slot (τ_1).

We refer to the source of the weaker link as *the node* in this derivation for clarity purpose. Under an idle slot, the node may be decrementing its backoff or experiencing an EIFS wait period. We also assume that the DIFS period (which is realistically around $50\mu s$) is zero since it is greatly lesser than the EIFS period (around $380\mu s$). Simulation validation shows that this approximation is reasonable.

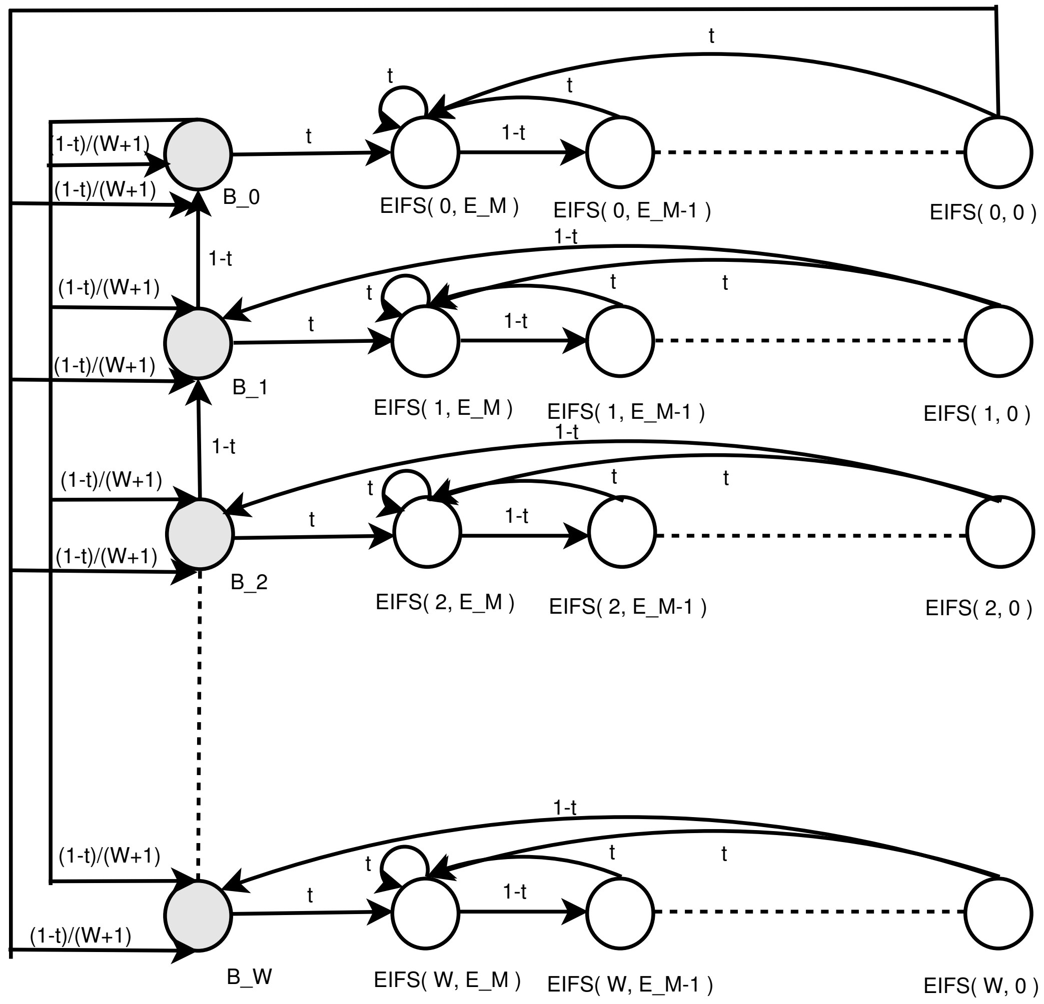


Figure 5: Markov chain for EIFS calculation

In order to compute the state space, we observe that the source may be decrementing its backoff or experiencing an EIFS wait period during an idle slot. The i^{th} backoff stage is denoted by $B(i)$ where $0 \geq i \geq \text{CW}_{\min}$. We represent the EIFS duration by M slots where slot j represents the number of slots left for completion of the EIFS duration. $E(i, j)$ denotes the j^{th} EIFS slot during the i^{th} backoff stage and $0 \geq j \geq M$. $B(i)$ and $E(i, j)$ are the states of the chain. The chain is represented in Figure 5. In this figure, the variable τ_2 is represented as t.

The channel becomes busy for the weaker link when the stronger link starts transmitting during an idle slot (τ_2). The value of τ_1 is dependent upon τ_2 since the weaker link experiences greater EIFS related backoffs when τ_2 is higher. However, τ_2 is independent of τ_1. The transition probabilities between the states are represented in Table 3 and are calculated based on the following set of rules.

During the backoff period, a node will move from backoff stage $B(i)$ to backoff state $B(i-1)$ when the channel is sensed idle at the end of a slot (Rule 1). If the channel is sensed busy, it will freeze the backoff and start its EIFS (at state $E(i, M)$) once the channel becomes idle again (Rule 2). While in EIFS, the node will

Rule	From	To	Probability
1	B(i), $i \neq 0$	B(i-1)	$1 - \tau_2$
2	B(i)	E(i,M)	τ_2
3	E(i,j), $j \neq 0$	E(i,j-1)	$1 - \tau_2$
4	E(i,j)	E(i,M)	τ_2
5	E(i,0), $i \neq 0$	B(i)	$1 - \tau_2$
6	B(0)	B(i)	$\frac{1-\tau_2}{\text{CW}_{\text{min}}+1}$
7	E(0,0)	B(i)	$\frac{1-\tau_2}{\text{CW}_{\text{min}}+1}$

Table 3: Transition probabilities

decrement the number of EIFS slots to wait if the channel is sensed idle (Rule 3). If the channel becomes busy during an EIFS, the node will resume EIFS from the start when the channel is sensed idle again, hence moving to the state $E(i, M)$ (Rule 4). When the backoff stage reaches 0 (stage B(0)), the node will transmit the packet and then choose a uniform random backoff from $[0, \text{CW}_{\text{min}}]$ upon the successful completion of DIFS(Rule 6). Similar explanations can be provided for the other rules.

The node starts transmitting the packet only when the channel is idle at the slot boundary when: (1) the backoff counter is zero (state $B(0)$); or (2) The EIFS period is completed and backoff counter is zero (state $EIFS(0, 0)$). Hence, the probability with which the node starts transmitting a packet at an idle time slot (τ_1) is given by Equation 24.

$$\tau_1 = (1 - \tau_2)(\Pi_{B(0)} + \Pi_{EIFS(0,0)}) \tag{24}$$

where Π are the limiting probabilities of the above chain.

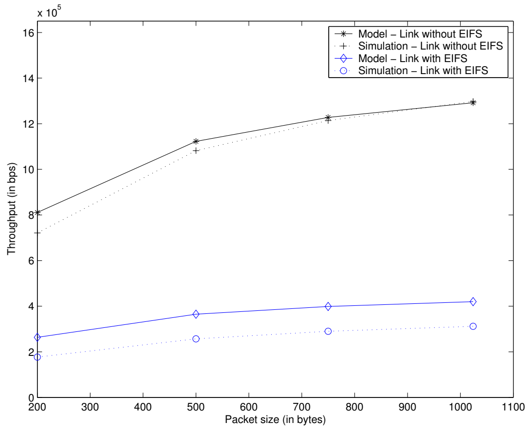


Figure 6: Throughput study for EIFS effect

Figure 6 validates the model by comparing it with simulation (with standard MAC parameters). The simulation was conducted using the QualNet simulator, which implements a detailed model of IEEE802.11 [12]. Packet size was varied from 200 bytes to 1024 bytes. Since the links compete with a ratio $\tau_1 : \tau_2$, a constant ratio of the throughput between the weak and the strong link can also be seen (24% according to the simulations and 32% according to the model). The comparison between the model and the simulation the indicates that the assumptions of the model (especially the discrete EIFS slots and independence of τ_1) are reasonable.

We now model the categories with hidden terminals (AIS, SIS and IDIS). The models include the impact of the backoff mechanisms in a manner that allows different strategies to be evaluated. We first derive a general throughput estimation model and discuss the effect of hidden terminals with respect to this model.

6.2 General Hidden Terminal Scenario

In this section, we derive a generic model to compute the *long-term* throughput of the links under hidden terminals using *Renewal Reward Process*. We then specialize this model to account for the different interaction cases. Figure 7 shows the abstraction of the events observed at a source between two successful packet transmissions.

6.2.1 Modeling long-term throughput as a Renewal Reward process

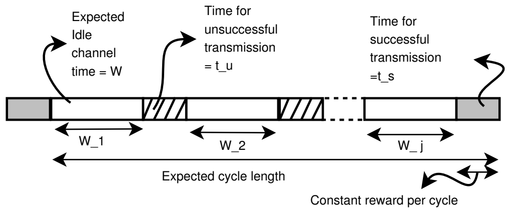


Figure 7: Packet transmission attempts

Consider the process of a source transmitting a packets. Let t_s and t_u represent the constant packet transmission durations for a successful and unsuccessful attempt, respectively. The source waits for a certain amount of time when the channel is idle (to decrement its backoff) and transmits the packet. The probability that a link starts transmitting at an idle slot is denoted by τ (conditional transmission probability) [2]. The packet may be successfully transmitted or may lead to a collision. Let p represent the packet loss probability given that the link transmitted a packet (conditional collision probability [2]). Let W_i be the random variable denoting the wait times before the source transmits the packet. Let U_j be the number of attempts before successfully transmitting a packet. We assume the following (1) W_i are iid random variables; (2) The transmission initiation (transmit or not transmit in a given timeslot) and its result (success or collision) are Bernoulli trials; (3) W and U are independent. Under these assumptions, the long-term expected value of W is given by Equation 25. The expected value of U is given by Equation 26.

$$E[W] = \frac{1}{\tau} \tag{25}$$

$$E[U] = \frac{1}{1-p} \tag{26}$$

Consider the process where a source waits for a certain amount of time (W_i) and transmits a packet. Consider a renewal process which constitutes of each cycle ending with a successful transmission. Figure 7 shows one such cycle. Let $W_1, W_2, \ldots, W_i$ denote the wait times before each transmission and let $U_j = u$

represent the number of attempts before a successful transmission in one such cycle. We now find the expected value of the time required to complete one cycle (one successful transmission) given that $U_j = u$. We note that $U_j \geq 1$.

$$
\begin{aligned}
E[\text{cycle length}|U_j = u] = &\quad E\Big[\big(\sum_{i=1}^{u}(W_i + t_u)\big) - t_u + t_s\Big] \\
= &\quad \Big(\sum_{i=1}^{u}(E[W_i] + t_u)\Big) - t_u + t_s \\
= &\quad u(E[W_i] + t_u) - t_u + t_s
\end{aligned}
$$

Now, the expected value of the cycle length is given by:

$$
\begin{aligned}
E[\text{cycle length}] = &\quad E\big[E[\text{cycle length}|U_j = u]\big] \\
= &\quad E[u](E[W_i] + t_u) - t_u + t_s \quad \text{(since } W \text{ and } U \text{ are independent)}
\end{aligned}
$$

We now apply the renewal-reward theory to predict the long-term throughput. The expected reward per cycle is the number of payload bits transmitted in one cycle which is equal to Ct_s. Hence, the long-term throughput is given by Equation 27.

$$
T_i = \frac{\text{Expected reward per cycle}}{E[\text{cycle length}]} \tag{27}
$$

$$
= \frac{Ct_s}{n_u(t_w+t_u)-t_u+t_s} \tag{28}
$$

where $n_u = E[U]$ and $t_w = E[W]$ as given by Equations 25 and 26. The variables that need to be computed are p and τ, which vary based on the type of hidden terminal. The time required to transmit the DATA packet for link i is represented as l_i. We assume that both the links have the same data packet size.

6.3 AIS formulation

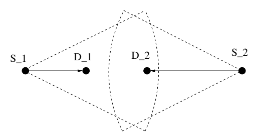


Figure 8: Hidden Terminals in AIS

Recall that in AIS, a source of one link can cause collision at the destination of the other, but not vice versa. Figure 8 shows this case where a transmission from S_1 can cause a collision at D_2. The transmission of S_2D_2 will succeed only during the idle periods of the link S_1D_1. This can lead to severe long term unfairness for S_2D_2.

We explain the derivation with respect to the scenario in Figure 8. The estimates for link S_1D_1 is straight forward since the link does not experience any hidden terminals. Hence, the value of $p_1 = 0$ and the value of $\tau_1 = \frac{2}{\text{CW}_{\min}}$.

Let p_2 and τ_2 represent the p and τ for link S_2. The link S_2D_2 can transmit only when S_1D_1 is not active, otherwise the transmission from S_1 will cause a packet collision at D_2. Let p_2 be the probability that the packet transmitted by S_2 will result in a collision conditioned on S_2 transmitting a packet. Let τ_2 be the probability that link S_2 starts transmitting at an idle slot.

Deriving p_2:

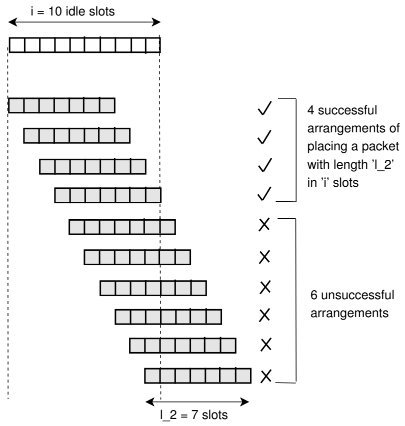


Figure 9: Packet success in AIS. i represents the CW chosen by S_1D_1 and l_2 denotes the packet length of the link S_2D_2

We first derive the success probability $(1 - p_2)$ of link S_2D_2. The packet transmission of S_2D_2 is successful only if the complete packet of S_2D_2 is transmitted when S_1 is inactive. A single slot of overlap between S_1D_1 and S_2D_2 can cause a packet collision at D_1. Let i be the congestion window (CW) chosen by the link S_1D_1. The link S_2D_2 can be successful only if the complete transaction of S_1D_1 lies within that duration of i slots. For example, as shown in Figure 9, let S_1D_1 choose a backoff of $i = 10$ slots and let $l_2 = 7$ slots. A transmission of S_2D_2 will succeed only if the 7 slots of transmission lie within the 10 slots when S_1D_1 is idle. As seen from the Figure 9, there are 4 possible arrangements of a successful transmissions out of 10 possible ways.

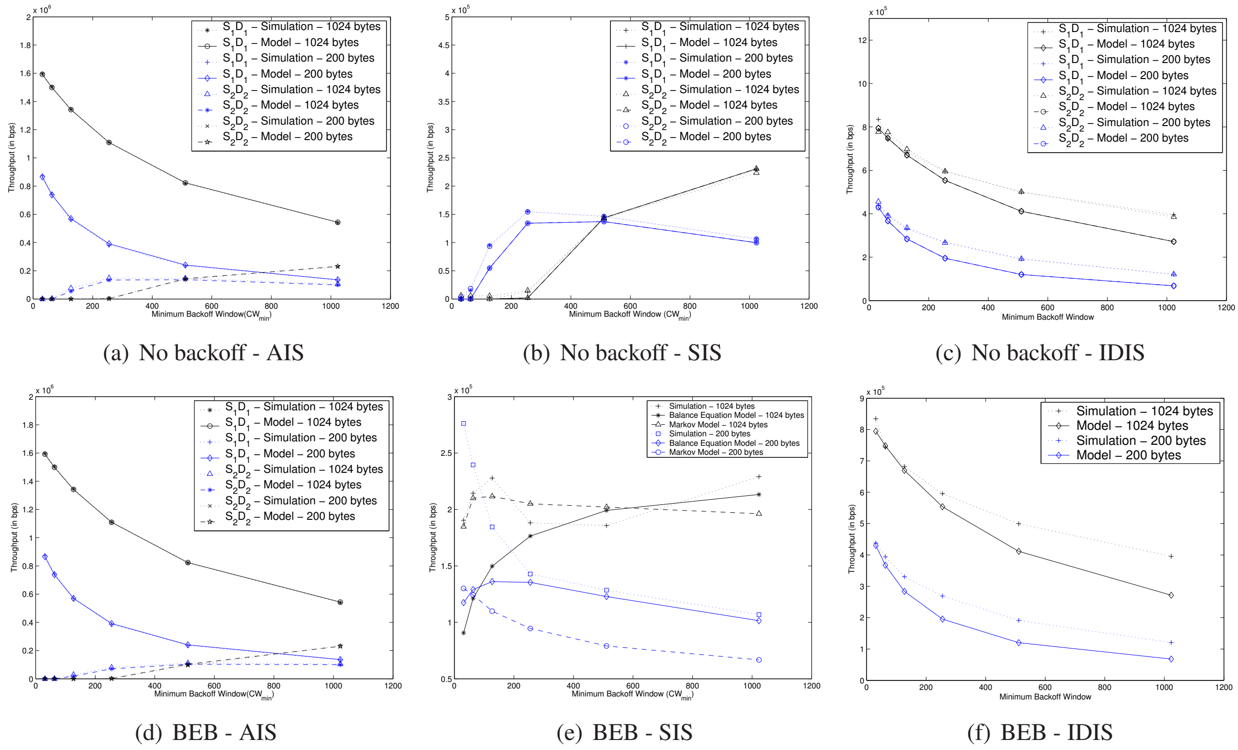
Generalizing this *arrangement* of l_2 slots in i slots of idle period, it can be shown that there are $(i-l_2+1)$ ways of placing a successful transmission from S_2D_2. Let $p'_2(i)$ be the probability that the transmission from link S_2D_2 succeeds given that S_1D_1 has chosen a backoff window of i. Equation 29 gives $p'_2(i)$ based on the number of successful *arrangements* of the transmission.

$$\bar{p}_2(i) = \begin{cases} 0, & \textit{if } i < l_2 \\ \frac{(i-l_2+1)}{i}, & \textit{otherwise.} \end{cases} \tag{29}$$

Since the probability of choosing i from $[0, \mathrm{CW}_{\min 1}]$ is $\frac{1}{\mathrm{CW}_{\min 1}+1}$ it can be shown that:

$$p_2 = 1 - \frac{\sum_{i=0}^{\mathrm{CW}_{\min 1}} \bar{p}_2(i)}{\mathrm{CW}_{\min 1} + 1} \tag{30}$$

Under the BEB scheme, the value of τ_2 can be calculated by Equation 22. We also model AIS throughput under a simple scheme where backoff window is always chosen from 0 to $\mathrm{CW}_{\min}$ irrespective of the collision of the transmitted packet (which we refer henceforth as *No backoff* mechanism), $\tau = \frac{\mathrm{CW}_{\min}}{2}$. The comparison of BEB model with this model helps to identify the effectiveness of BEB.



(a) No backoff - AIS (b) No backoff - SIS (c) No backoff - IDIS

(d) BEB - AIS (e) BEB - SIS (f) BEB - IDIS

Figure 10: Effect of Hidden terminals

This completes the calculation of all the variables (p's and τ's) for throughput estimation of the links.

We now compare the effectiveness of AIS throughput formulation. Under standard MAC parameters (with $\mathrm{CW}_{\mathrm{min}} = 31$), the link with the hidden terminal cannot successfully transmit (even relatively smaller packets) between the idle time of the other link (because $\mathrm{CW}_{\mathrm{min}}$ is only 31 slots). Hence, we vary the $\mathrm{CW}_{\mathrm{min}}$ of the links and validate the model for varying $\mathrm{CW}_{\mathrm{min}}$ and packet sizes.

As seen in Figure 10(d), the prediction by the model matches closely with the simulations. It can be seen that the starving link S_2D_2 gets a fair throughput only when $\mathrm{CW}_{\mathrm{min}}$ is very high. Exponential backoff at S_2 will reduce the frequency of transmission of link S_2D_2. However, the interfering traffic at S_1D_1 is at constant rate and exponential backoff does not improve the success probability of S_2D_2. This absence of correlation between the change of interference pattern over time makes BEB ineffective in AIS. It can be seen that the link S_2D_2 will get zero throughput until the $\mathrm{CW}_{\mathrm{min}}$ (of S_1D_1) value is large enough to accommodate the the packet. This suggests that under low $\mathrm{CW}_{\mathrm{min}}$, the effect of asymmetric hidden terminals can be reduced by either decreasing the packet size (or increasing the transmission rate). These parameters can be calculated directly from the model.

6.4 Preliminary Formulation for Symmetric categories

Symmetric hidden terminals occurs in the SIS and IDIS categories. Computation of the throughput variables p and τ for symmetric categories is hard due to the coupling between the two links. This makes independence assumptions on the probability of collision per backoff stage inaccurate. An accurate model of these cases would require modeling the combined states of the two senders (each of which may take any of the states in the Bianchi model), leading to a very large Markov chain. Nevertheless, we present results with the approximate model. We believe that an accurate model of SIS cases is an open question that deserves a more thorough treatment.

SIS formulation:

In SIS, the source of a link (S_1 or S_2) causes collision of the packet at the destination of the other link (D_2 or D_1). Reception at D_2 is successful if S_1 does not transmit in slots that overlap with S_2's transmission. Due to symmetry of the hidden terminals, we have $p = p_1 = p_2$ and $\tau = \tau_1 = \tau_2$.

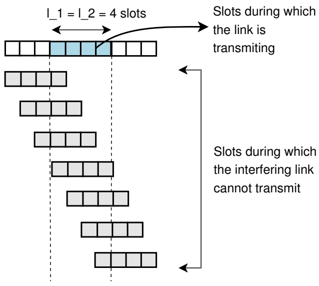


Figure 11: Packet success in SIS. l_1 and l_2 represents the slots required to transmit the packet for link S_1D_1 and S_2D_2

We now derive the the success probability $(1 - p)$. If the value of maximum backoff is lesser than the time required to complete a successful transmission (and ACK), then S_2D_2 cannot find a sufficiently long gap between S_1D_1's transmission and hence $p = 0$. Otherwise, the packet will be transmitted successfully if the interfering link does not transmit a packet such that atleast one slot overlaps with the packet transmission. For example, Figure 11 shows the transmission of the data packet by one link (say S_1D_1) in light blue. The slots during which interfering link (S_2D_2) cannot transmit is colored in grey. If τ is the probability that the interfering link will transmit in a given time slot, then the probability that it will transmit in the slots that will collide with the given packet is given by Equation 31.

$$\begin{aligned} p = &\quad \tau + (1-\tau)\tau + \ldots + (1-\tau)^{\lceil 2l \rceil - 1}\tau \\ p = &\quad 1 - (1-\tau)^{2\lceil l \rceil} \end{aligned} \tag{31}$$

The relationship between p and τ is given by the Equation 22 which we term $\tau = b(p)$.

Symmetric hidden terminals have the property of one link being affected by the activity of the other. The probability of drop on link S_1D_1 (p_1) depends on the frequency of packet transmission attempts at link S_2D_2 (τ_2). Let us denote this relation by $p_1 = f(\tau_2)$. Owing to symmetry in the topology, we can represent the above by two relations: (1) $\tau = b(p)$; and (2) $p = f(\tau)$. Equation 31 can be used as an approximation for representing the function $p = f(\tau)$. The roots of the above equations can be calculated by standard numerical techniques like Newton method [11]. Improving the expression for f from that in Equation 31 is a part of our future work.

Integrating the existing components for calculating p and τ was also attempted. A Markov chain based approach to calculate τ was proposed in the study [7]. Figure 10(e) study the throughput of the links when the $\text{CW}_{\min}$ and packet sizes are altered. It compares the simulation with the two models (the Garetto model and the one proposed here). Our model matches the simulation only under higher $\text{CW}_{\min}$ values. The Markov-chain based model captures the throughput trend for larger packet sizes, while a large gap exists under lower packet sizes. We believe that an accurate model of SIS remains an open issue. The model is accurate for No-Retry mechanism(Figure 10(a)). At lower values of $\mathrm{CW}_{\min}$, BEB scheme outperforms the No-retry scheme. The exponential backoff of one link helps to create enough channel idle time for packet transmissions of the the other link. However, such a scenario exhibits short term unfairness where the throughput of one link dominates for short periods of time.

IDIS formulation: Recall that in IDIS, only receivers are in interference range with each other. A receiver can cause a drop on the other link when it transmits an ACK. Due to the symmetry of the topology, p and the τ are identical for the two links, but are coupled. Their value can be derived in a method similar to symmetric hidden terminals, under the same imperfect assumptions. The results of this model are shown in Figure 10(f).

7 Extensions and Applications of the Models

The next steps in this research are to pursue further generalizations and applications of the interference models. We are working on a generalization that takes into account an SINR physical model. We are also working on traffic engineering and QoS models, as well as distributed routing protocols that incorporate accurate accounting for interference. In this section, we describe early experiences with another area of future extension that we think is promising: an analysis of the self-interference modes of interactions that arise in multi-hop chain connections.

Links in a chain topology can exhibit the different modes of interference that were discussed in the paper, leading to significant impact on the expected performance of these chains (without considering the impact of interference across chains). However, given the restrictions of chain connectivity the probability of the different cases change. Concurrently, different links may exhibit different relationships with other links. For this reason, and given that downlink links receive their packets from links closer to the source, separate throughput models are required.

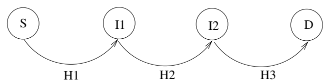


Figure 12: A Chain with 3 hops

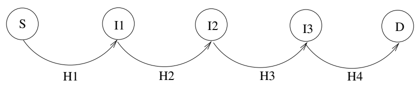


Figure 13: A Chain with 4 hops

Figure 12 shows a chain with 3 hops. In this chain hops H1 and H3 are two link level flows within this chain that interact with each other according to the probabilities shown in 14.

It can be seen from the plot that at typical interference range of more than twice the communication range, only SCSI interactions are possible. In a 4 hop chain as shown in Figure 13 there are three sets of two flows: H1 and H3, H2 and H4, H1 and H4. The first two of these interaction are similar to the 3 hop scenario and would belong to SCSI category for typical wireless network cards. The probabilities of types

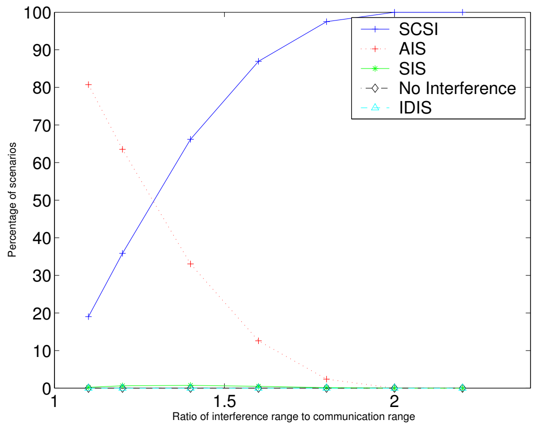


Figure 14: Probabilities of Interaction Categories for 3 Hop Chain

of integrations for this set is shown in figure 15 which shows that categories AIS and SCSI dominate this interaction.

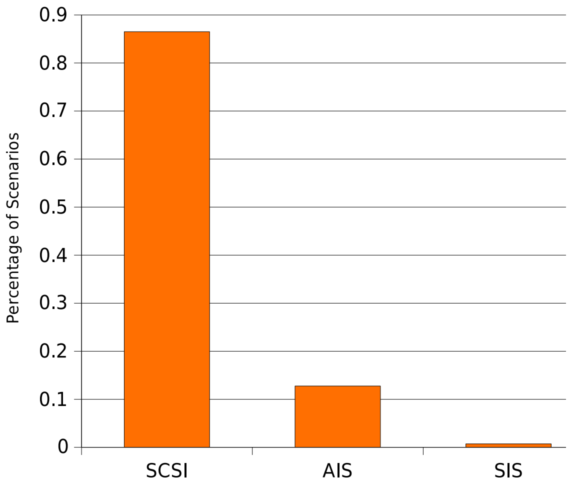


Figure 15: Probabilities of Interaction Categories for 4 Hop Chain

These results were obtained by Monte Carlo simulation of chain topologies with 4 hops. The graph shows that with 4 hop chains, there is a high probability that hops in a single chain will belong to the AIS group. The behavior of AIS and SCSI are very different and each will have an effect on the throughput achievable in a chain topology. Analyzing ways of detecting these situations and designing routing protocols that take advantage of this information is part of our future research.

Our most immediate future work include using the more realistic Signal to Interference and Noise Ratio (SINR) interference model in place of the two-disc model. We believe that the proposed geometric framework becomes more important as the number of possible interactions explodes under the SINR model (an estimated 20736 individual interactions between two-flows under the SINR model). In SINR model a receiver can successfully receive a packet from a sender in the presence of an interfering node as long as the distance between the sender and receiver is some factor shorter than the distance between the receiver and the interfering node. Based on this we can have many different interactions. The notation A-B-C specifies whether or not A can Capture B when B and C transmit together. There are 8 of these interactions that we are interested in:

1. S1-D1-S2
2. S1-D1-D2
3. D1-S1-S2
4. D1-S1-D2
5. S2-D2-S1
6. S2-D2-D1
7. D2-S2-S1
8. D2-S2-D1

Each one of these interactions can have 6 different states. For example S1-D1-S2 can have the following states:

1. S1 Captures D1 when S1 and S2 are in Communication Range
2. S1 Can not Capture D1 when S1 and S2 are in Communication Range
3. S1 Captures D1 when S1 and S2 are in Interference Range
4. S1 Can not Capture D1 when S1 and S2 are in Interference Range
5. S1 Captures D1 when S1 and S2 are Out of Range
6. S1 Can not Capture D1 when S1 and S2 are Out of Range

Since each of these 8 interactions can be in 6 different states, we have a total of 6^8 scenarios(more than 1.5 million). We notice that some of these interactions are not possible. Take for example the case of S1-D1-S2 where S1 Captures D1 when S1 and S2 are in Communication Range, for this interaction between S1-D1-S2, the interaction S2-D2-S1 can only have two states because S1 and S2 are in Communication range namely:

1. S2 Captures D2 when S1 and S2 are in Communication Range
2. S2 Can not Capture D2 when S1 and S2 are in Communication Range

With this restriction we notice that each interaction can have states as shown in Table 4

	Interaction	States
1	S1-D1-S2	6
2	S1-D1-D2	6
3	D1-S1-S2	6
4	D1-S1-D2	6
5	S2-D2-S1	2
6	S2-D2-D1	2
7	D2-S2-S1	2
8	D2-S2-D1	2
	Total	20736

Table 4: Table showing the number of states of each interaction

8 Concluding Remarks

The paper makes several contributions to the analysis of two single hop wireless flows. Specifically, it relaxes the assumption of a constant interference-range (also carrier sense range) to communication-range ratio in existing two-flow models, which does not hold in practical radios. Additional types of interactions occur under these assumptions, which the paper categorizes. It also develops closed form expressions for the probability of occurrence of the scenarios and analyzes their frequency as a function of the interference/carrier sense range. The paper also contributes constructive models for the throughput in presence of hidden terminals, although the models for SIS remain quite approximate.

Our most immediate future work includes generalizing the model to account for a more realistic physical environment based on Signal to Interference and Noise Ratio. In addition, we presented some early results of studying the impact of interference in a chain topology from first principles. We also seek to improve the throughput models we developed for symmetric hidden terminals which do not match simulation in all cases. Such a model will allow us to account for the effect of capture, and likely expose additional interaction cases more representative of realistic radio operation.

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