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## Modeling and analysis of two-flow interactions in wireless networks

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## ABSTRACT

Interference plays a complex and often defining role on the overall performance of wireless networks, especially in multi-hop scenarios. Understanding this role is critical for understanding these networks, and in turn for developing effective protocols for them. In the presence of interference, Carrier Sense Multiple Access MAC protocols are known to suffer from the hidden terminal and exposed terminal problems, which can cause poor performance and unfairness. Recent work has shown that depending on the relative location of interfering sources and destinations, several modes of interference exhibiting different behavior, occur. In this paper, we first relax the assumption that the interference range is equal to the reception range. This gives rise to a large number of interference configurations; we develop closed form expressions for their frequency of occurrence. As a result, we discover that the frequency of occurrence of the major modes of interference change significantly from those obtained without relaxing the interference range assumption. More importantly, we show that two previously unknown modes of interactions arise, whose performance differs significantly from the known modes. We develop models for estimating the throughput for the different categories of interaction, and validate them against simulation results. We believe that this analysis represents a further step into the understanding and characterization of the impact of interference from first principles.

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## 1. Introduction

In Multi-Hop Wireless Networks (MHWNs) that use Carrier Sense Multiple Access (CSMA) MAC protocols, different forms of hidden terminal and exposed terminal problems arise [1,2], which can lead to poor link quality, and short term or long term unfairness. Complex interactions occur between interfering links based on the relative location of the senders and receivers (more accurately the state of the channel between them). These interactions play an important role in determining the link quality, and can give rise to sustained or short term unfairness. Understanding these interactions is critical for understand-

ing and characterizing behavior in MHWNs and for designing effective protocols for them.

Recent work has analyzed and classified the different behaviors that arise between two interfering links that use the IEEE 802.11 protocol [3,2]. Geretto et al. show that each secondary link can be in two possible states (out of range, in range). Since there are four secondary links there are 16 ( $2^4$ ) possible scenarios. After removing the redundant cases there are 12 different interactions. In this work we allow the interference range to be different from reception range which results causes cross links to have three different states (out of range, in reception range, or in interference range). This causes the number of scenarios to  $3^4$  which after removing the redundant cases reduces to 53 different interactions.

Understanding and characterizing interactions at the MAC level is a promising first step towards an understanding of the effect of interference from first principles. This

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understanding can be helpful in determining the placement of nodes in a sensor networks where destructive interactions can be avoided. Especially in case of mobile devices, where the nodes causing destructive interactions can be moved to improve network performance. This study also helps us in setting network parameters like transmit power, carrier sense threshold and receiver sensitivity in a more informed manner. These parameters can be determined in order to reduce destructive interactions. Furthermore, the effect of these interactions within a multi-hop route between two nodes can be used as part of a routing metric aiding routing protocols in selecting more efficient links.

This paper makes several contributions for improving the analysis of two-flow interference, using more realistic assumptions, identifying additional types of interactions, and analytically modelling their behavior. Specifically, we make the following contributions:

- (1) *Generalizing the analysis by allowing Interference/Carrier Sense range being different from reception range (Section 5).* This generalization results in a large number of individual scenarios. Earlier works have shown 12 different interactions while our study identifies 53 different interactions which can be grouped based on the type of interaction into five different classes. This grouping helps in determining the behaviors of these interactions and helps in reducing the complexity of analyzing numerous individual interactions.
- (2) *Geometric analysis, leading to closed form expressions, for the probability of occurrence of the scenarios (Section 6).* In contrast to the existing geometric models [3], we use a new simpler approach that allows direct evaluation of the probability of the grouped cases (avoiding the need to model each of the individual 53 scenarios). The geometric models validate very well to a Monte Carlo characterization of the probability.
- (3) *Analytical models characterizing the performance of the different classes of interactions (Section 7).* The models are validated using simulation. This includes models for the two new identified classes of interaction.

We believe that these contributions collectively enhance the understanding of causes and impact of interference. Studying the behavior of two flow interactions is an important first step towards understanding complex interactions in a general network. We have used the results of this study to predict the behavior of multi-hop routes between source and destination in a multi-hop wireless network in the presence of interactions between hops of the same chain as well as interference between hops across chains [4,5]. Several important steps remains towards a generalization of this understanding, including the use of a more realistic channel model and experimental validation of the results. We present our conclusions and thoughts about future work in Section 8.

## 2. Related work

Carrier Sense Multiple Access (CSMA) [6,7] based protocols such as IEEE 802.11 are heavily used for medium access in wireless networks. CSMA works by attempting to prevent sources from transmitting concurrently by having each source sense the channel before transmission. In wireless networks, the state of the channel at the receiver is what determines whether a reception occurs successfully. As a result, carrier sense at the sender does not accurately reflect the state at the receiver. More precisely, if the receiver channel is busy but the sender channel appear idle, a collision occurs – *the hidden terminal problem*. Conversely, if the receiver channel is available but the sender channel appears busy, the transmission is unnecessarily deferred – *the exposed terminal problem*. These problems are known to significantly degrade the performance of CSMA in wireless settings [6,7].

A number of protocols have been proposed to attempt to reduce the effect of the hidden terminal and exposed terminal problems. Wu and Li propose using a busy tone channel that is used by the receiver when it is receiving a packet [8]. Other sources sense this channel allowing carrier sense based on the receiver position. Karn proposed the MACA protocol (Multiple Access Collision Avoidance), a predecessor to the current IEEE 802.11 protocol [9]. MACA uses short request-to-send (RTS, sent by the sender before transmission) and clear-to-send (CTS, sent by the receiver if RTS is received correctly and the channel is available) to attempt to reduce collisions. Potential interferers that receive the RTS or the CTS packet do not transmit for the duration of the packet. Bhargavan et al.'s MACAW protocol improves MACA by adding acknowledgement and retransmission [1].

Despite these advances in protocol design, the hidden terminal problem continues to plague CSMA MAC protocols. In fact, depending on the relative location of the senders and receivers, and other factors such as the MAC protocol, and the interference model, a number of interaction modalities with distinct behavior occur. Bhargavan et al. identify and discuss several of these cases and propose modifications to the MACAW protocol to address them individually [1] in a network where the interference range is equal to the reception range.

Our work is most related to the following two efforts that attempt to methodically characterize and analyze the performance of the different modes of interactions that occur between two interfering links. Rogers and Abu-Ghazaleh [2] conduct a simulation study of all the possible configurations of two interfering links under saturation traffic and with a fixed interference range which is significantly larger than the reception range. They enumerate the possible modes of interaction, and discover a number of cases with destructive interactions both with RTS/CTS and without RTS/CTS.

Most relevant to our work, Garetto et al. enumerated the types of interactions that occur under assumptions of transmission range equal to interference range, and developed geometric models for analyzing their expected frequency [3]. They also presented analysis the performance

using simulation. They grouped the scenarios into three general classes based on their behavior. We generalize their analysis in a number of important ways, discovering two new modes of interactions, which leads to a more realistic characterization of the impact of interference on CSMA protocols. Their assumption that transmission range is equal to interference range restricts the number of interaction categories that they observe. Relaxing this assumption allows us to discover new interaction categories that have significantly different performance. Moreover, their work presents geometric modelling of each interaction which does not scale for a larger number of interactions that exist for more realistic models of packet reception. We have grouped similar interactions into distinct categories and then developed geometric models for each category.

In our previous work [10] we generalize the work in [3] to include cases where interference range is more than communication range. This generalization produces new cases that were not possible before. In this paper we present a detailed derivation of the probability of occurrence of all five cases. We also develop and present throughput models for these five cases. In [11] we derive the interactions using the SINR model (whereas in this paper we derived them for the two disc model). The derivation and results are quite different: the SINR case is significantly more complicated, leading to more than 1 million individual cases. This forces us to use approximations and heuristic estimates. In contrast, in the current paper, the simpler model allows much cleaner derivation and closed form results. Finally, our work [11] showed that the benefits from using the SINR model are small and the two models are quite close; the additional complexity does not expose significantly different results. Thus, the simpler and cleaner two disc model presented in this paper is sufficient for most cases, and is probably more appropriate (because of complexity) for forming practical protocols.

Models for computing throughput in CSMA networks were studied initially by Boorstyn et al. [12] and Tobagi et al. [13]. Advanced models for calculating the throughput in IEEE 802.11 based networks have been proposed [14–17]. Even though these works account for the effect of interactions, they do so for specific networks and using iterative methods. In contrast, the focus of the paper is to classify and analyze all the possible interactions between two contending links and to model the resulting behavior constructively from first principles, based on the interactions that cause them.

### 3. Background

In this section, we briefly overview the channel access mechanism in wireless CSMA MAC protocols such as the IEEE 802.11. The power of a transmitted packet degrades with distance, obstacles, and other wireless propagation effects that arise due to the signal interaction with the surrounding environment. In the absence of interference, a packet is successfully received if the signal strength at the receiver is above the receiver sensitivity threshold. Furthermore, when there is interference, a node can receive a

packet successfully if the signal to interference and noise ratio (SINR) is above the capture threshold.

A commonly used simplifying model of reception and interference is the *physical model* [18] where a transmission from a node can be sensed by all the nodes that are within a given *Interference Range* ( $R_i$ ). In the absence of interfering signal, a packet can be received by all the nodes that are within the *Communication Range* ( $R_c$ ) ( $R_c < R_i$ ). A receiver that is within  $R_i$  but outside the  $R_c$  can sense the channel as busy, but will not be able to decode the signal. Under such a model, packet collisions occur when a node is receiving a packet and an interfering node (that is within a distance of  $R_i$  from the node) transmits a signal. The model is approximate because it does not take into account the capture threshold.

The MAC protocol attempts to regulate access to the channel to reduce collisions. In wireless networks, carrier sense is less effective than in wired networks because the state of the channel at the sender (where sensing is carried out) is different from its state at the receiver (where a collision can occur). Furthermore, it is possible for interferers that are outside communication range to interfere with a packet (they cause the SINR to drop below the capture threshold); this is the primary reason why the RTS/CTS mechanism in IEEE 802.11 is not very effective. As a result, most wireless cards use a conservative carrier sense threshold to reduce the effect of hidden terminals. Our work generalizes the previous models of two flow interactions to take into account the impact of the carrier sense/interference range.

IEEE 802.11 is a CSMA/CA based MAC protocol that is prevalent in today's wireless networks. The *basic mode* of IEEE 802.11 consists of a DATA-ACK handshake, where the DATA packet is transmitted by the sender after carrier sensing and an acknowledgement (ACK) packet is sent back by the receiver if the packet is successfully received. An optional RTS/CTS handshake can also be used in 802.11 protocol. The sender first sends a *Request to send* (RTS) packet and the receiver signals its readiness to accept by sending a *Clear To Send* (CTS) packet. The DATA-ACK handshake proceeds after a successful RTS/CTS exchange.

In order to foster fair access to the channel, the sender maintains a backoff window (BO) counter. Initially, the sender sets the CW a minimum value  $CW_{min}$  and chooses a random number between 0 to CW. If the channel is sensed idle for *DCF Inter-frame Space* (DIFS) amount of time (DIFS is a 802.11 specific parameter), then the value of BO is decremented after every *slot time* (another predetermined unit of time used in IEEE 802.11) of channel idle time. The packet is transmitted when the value of BO becomes zero. When a collision is detected, the sender will double its value of CW and the backoff procedure repeats. Upon successful transmission of a packet, the sender resets its CW to  $CW_{min}$ . This process is called *Binary Exponential Backoff* (BEB). The maximum value of CW is bound by another threshold  $CW_{max}$ .

Another mechanism to reduce the greedy channel access is the use *Extended Inter-frame Space* (EIFS), which is a MAC specific parameter ( $EIFS \gg DIFS$ ). If a corrupted packet is received at a node, then it assumes an interfering traffic. In order to avoid interfering with this transmission,



the node decides to wait for longer duration of time (approximately till the interfering traffic has received an ACK) before it starts to decrement its backoff (instead of starting it after DIFS period). This wait time is denoted by a EIFS MAC parameter. If the node listens to an ACK packet, then it assumes the completion of the interfering traffic and resets the wait time back to DIFS.

#### 4. Overview of existing model

Garetto et al. [3] categorize the two-flow interactions using a boolean physical model where the transmission radius is equal to the interference radius. In this scenario, the nodes for each of the four secondary link can be either in range or out of range, leading to  $2^4$  different scenarios corresponding to the different combinations of states that each of the four links can be in. The 16 scenarios are reduced to 12 by eliminating the dual scenarios (scenarios that are identical other than relabelling the connections). They compute the occurrence probability of each of the scenarios conditioned on a fixed distance between the primary senders and receivers. More interestingly, they recognize that the individual scenarios can be grouped into three basic categories described below.

- (1) *Sender-Connected (SC)*: This category includes all scenarios where the two senders are within interference range. In SC scenarios, a sender will not start a transmission when the other sender is active due to CSMA and no collisions other than those when the two senders start transmission at the same time will occur (such collisions are of low probability due to the randomization of the backoff period incorporated by IEEE 802.11). Fig. 2 shows an example SC scenario.
- (2) *Asymmetric Incomplete State (AIS)*: In the remaining scenarios the senders are not connected (Incomplete State). A distinguishing attribute for the remaining scenarios is whether the state of the  $S_1, D_2$  and  $S_2, D_1$  states are identical (Symmetric) or different (Asymmetric). In Asymmetric Incomplete State, only one of the senders can interfere with the other destination. Thus, only one of the flows experiences packet collisions. Fig. 4 shows a sample scenario for AIS where the flow ( $S_2, D_2$ ) experiences a packet collision from  $S_1$ . An *incomplete state* is created since the source of the weaker link ( $S_2$ ) does not have complete information about the channel at its destination ( $D_2$ ). Having a complete information could have prevented the packet collision for ( $S_2, D_2$ ).
- (3) *Symmetric Incomplete State (SIS)*: Under this category, the senders are not connected. However, either both the senders can interfere with the other destination, or they cannot. In these scenarios, short term unfairness may arise, but no bias exists to lead to long term unfairness between the connections. Fig. 5 shows a sample scenario for SIS where the flow ( $S_1, D_1$ ) and ( $S_2, D_2$ ) interfere with each other.

#### 5. General two-flow interactions

In a wireless network, the state of the channel at the receiver determines whether a reception occurs successfully or not. However, carrier sensing is performed at the sender in Carrier Sense Multiple Access (CSMA) protocols. Accordingly, if the receiver channel is busy but the sender channel appears idle, a collision can occur. The geometry of the interfering links (more accurately, the state of the channels between them) determines what MAC level interactions arise. These interactions can significantly impact performance or cause short term or long term unfairness.

Given two interfering links  $S1-D1$  and  $S2-D2$ , the type of MAC interaction that occurs depends on the state of the secondary (or unintended) channels between  $S1-S2$ ,  $S1-D2$ ,  $S2-D1$  and  $D1-D2$  as shown in Fig. 1. Each of these channels can be in a number of states (in reception range, in carrier sense range, in interference range, or in interference range with capture), resulting in a large number of interaction types.

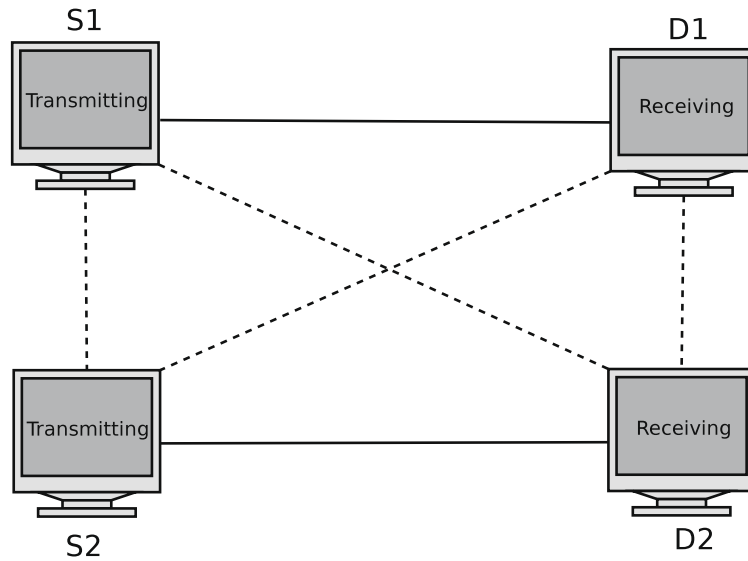
This section presents the categories of interaction that arise when we relax the assumption of the interference/carrier sense range being equal to the communication range. Specifically, we assume that a sending node  $S_1$  causes a collision at a receiving node  $D_1$  if they are within interference range with each other. As a result, the possible states of the four secondary flows ( $S_1S_2$ ,  $S_1D_2$ ,  $D_1S_2$  and  $D_1D_2$ ) now become: (1) in communication range; (2) in interference range, but not in communication range; (3) out of range.

Each of the four cross links can be in one of the above three states relative to each other for a total of  $3^4$ , or 81 enumerable scenarios. After removing the dual scenarios which are identical other than relabelling of the connections, a total of 53 distinct scenarios remain. For example the scenario where  $S_1D_2$  in interference range with all other links out of range is the dual of the scenario where  $S_2D_1$  is in interference range with all other links out of range.

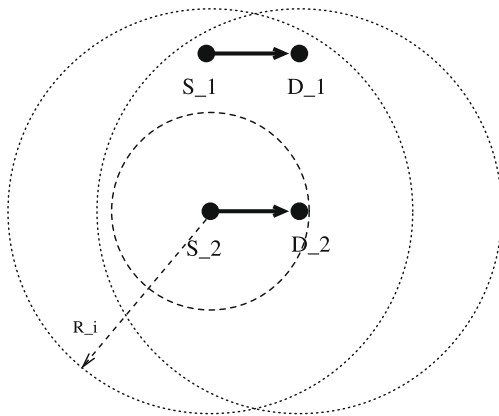
Luckily, the scenarios share important characteristics that allows us to classify them into a small number of categories (five in this case). We call scenarios where the two senders are within interference range (including those within communication range) *Sender Connected*. In such scenarios, the two senders arbitrate the channel successfully and fairly. The term *Symmetric* is used when each of flow interacts with the other flow symmetrically; in other words, the state of the  $S_1D_2$  link is identical to the  $S_2D_1$  link. Like before, we use *Incomplete State* to mean that the senders are not connected.

Due to the possibility of links being in interference but not communication range, the categories of scenarios exhibiting different interference behavior grow from three to five. In the following we discuss the five categories in more detail.

- (1) *Senders Connected Symmetric Interference (SCSI)*: SCSI represents sender connected scenarios where there is symmetric interference between opposite source and destination. For example, if link  $S_1D_2$  is in



**Fig. 1.** Interaction between two flows. Solid lines show primary connections, while dotted lines show secondary (unintended) channels.

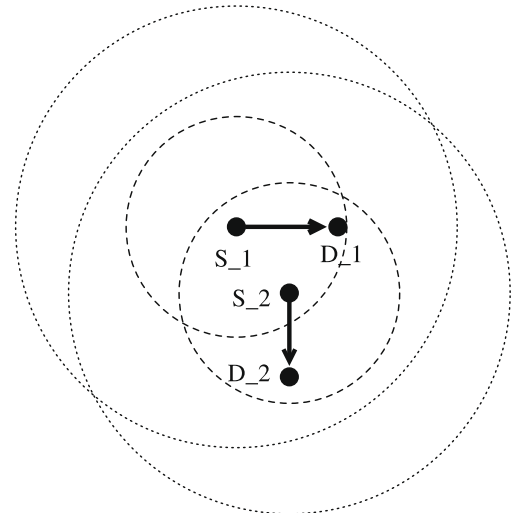


**Fig. 2.** Sample scenario for SCSI.

interference range then  $D_1S_2$  is also in interference range. Fig. 2 shows a sample SCSI scenario. Flows in this group share the medium fairly due to symmetry.

- (2) *Senders Connected Asymmetric Interference (SCAI)*: This subset of scenarios represent the first new category of interaction that we identify. In SCAI: (1) the senders are within communication range of each other; (2) One sender and the opposite receiver (belonging to the other flow) are in interference range (say,  $S_1D_2 \leq R_i$  in Fig. 3); and (3) The other sender and receiver are not in interference range of each other.

Fig. 3 shows a sample scenario in SCAI where  $S_1$  and  $D_2$  are in interference range, but not in communication range. Under the IEEE 802.11 protocol, the source  $S_1$  can sense the channel busy when  $D_2$  sends an ACK packet to  $S_2$ , but cannot decode the packet. It perceives such busy signal as an ongoing transmission. In order to avoid a possible collision,  $S_1$  will wait for the channel to be idle for EIFS period (a sig-



**Fig. 3.** Sample scenario for SCAI.

nificantly larger period than the standard DIFS inter-frame separation to ensure completion) before it transmits a packet.

$S_2$  on the other hand receives the ACK properly from  $D_1$  and will only wait the DIFS duration (which is  $\ll EIFS$ ) before decrementing its backoff. Since its backoff timer is much shorter than EIFS timer,  $S_2$  gets an unfair advantage in channel contention, wins the channel again and the cycle continues. This causes severe unfairness in two links and link  $S_1D_1$  starves.

- (3) *Asymmetric Incomplete State (AIS)*: This scenario is identical to the AIS category in classified in Garetto's work [3]. Briefly, in these scenarios, one of the flows (say,  $(S_1, D_1)$ ) can cause a packet collisions to the other flow but not vice versa. Thus, (1) the senders are out of range and can transmit simultaneously; and (2) One source and the opposite receiver are in interference range of each other and (3) The second

source and its opposite receiver are out of range of each other. Fig. 4 shows a sample AIS scenario. Many of the packets sent to  $D_2$  are lost because of interference from  $S_1$ , while  $D_1$  receives all packets from  $S_1$  successfully.

- (4) *Symmetric Incomplete State (SIS)*: The senders are out of range and both sets of opposite source and destination are within communication or interference range. Fig. 5 shows a scenario with SIS. This problem causes the overall throughput of the links to decrease substantially without affecting the fairness issue. Since the two senders are out of range, they will transmit simultaneously. Since each destination can be interfered by the opposite source, there is a packet drop at both the destination. This will cause significant low throughputs at for both links.
- (5) *Interfering Destinations Incomplete State (IDIS)*: This is the second newly identified category of interactions which is a subset of the originally classified SIS cases. This group includes scenarios where all the secondary links are out of range except the two destinations. Fig. 6 shows one such scenario. Since both the sources are out of range (not sender connected), they transmit packets simultaneously. The destination that receives its packet sends an ACK, thus causing a collision for the ongoing packet transmission at the other destination. This causes short term unfairness for each link. IDIS is a *Sender Unconnected, Symmetric* and *Incomplete state* scenario that experiences drops due to ACK-DATA packet collisions.

Scenarios belonging to IDIS and SIS groups are categorized as SIS group in Garetto et al. [3]. However, owing to difference in the interactions and throughput of the links, we classify them into separate groups. In SIS, since the

two sources will backoff at different levels independent of each other, the only opportunity for a source to successfully send a packet is when the source starts and ends the transmission within the backoff period of the other source. This small window for successful transmission would allow for better throughputs for shorter packet (smaller in size or higher rate). This behavior is different from IDIS where the links will have short term unfairness regardless of packet size. Fig. 7 compares the throughput of SIS and IDIS. As we increase the packet size the throughput of SIS group decreases while that of IDIS group remains unchanged (Fig. 7a). As the transmission rate is increased,

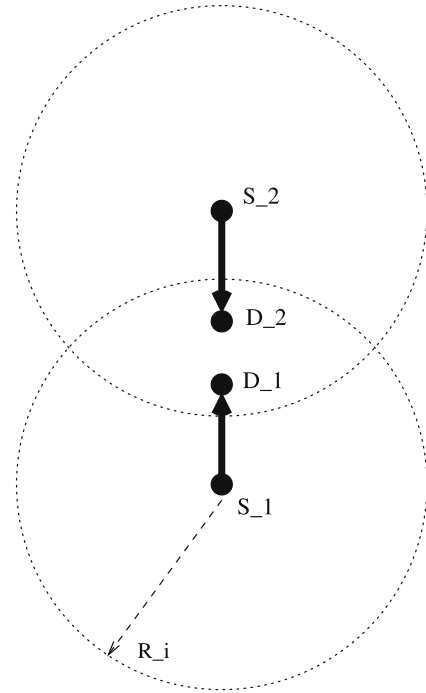


Fig. 5. Sample scenario for SIS.

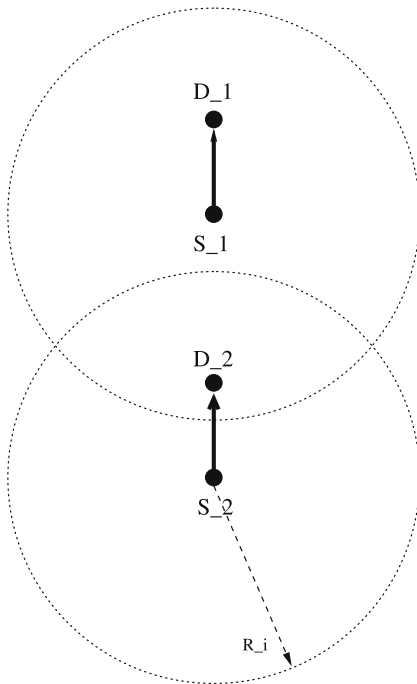


Fig. 4. Sample scenario for AIS.

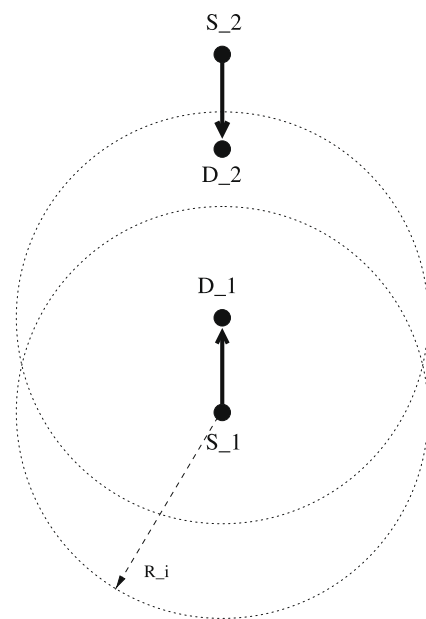


Fig. 6. Sample scenario for IDIS.

the amount of time required to send the packet reduces. Hence throughput of SIS increases (Fig. 7b).

The above interactions do not consider the use of RTS/CTS. With RTS/CTS, additional modes of interaction arise since RTS/CTS can be received by nodes in communication range, but not those in interference range. However, in practice most IEEE 802.11 networks disable the RTS/CTS option since it cannot prevent many interferes that are outside of communication range [19]. Thus, we decided not to pursue the additional interactions that arise in that mode.

## 6. Determining scenario probability

In this section, geometric models are developed to predict the probability of occurrence of the scenario groups identified in the previous section. Due to the increased number of cases, and the increased complexity of each case due to the addition of a separate interference range, we develop a new and simpler approach than the one used by Garetto et al. [3]. The problem is one of estimating different regions of intersection of the circles forming the communication and interference ranges of the different nodes, which correspond to the interaction scenarios. The Garetto approach would require a complex case by case treatment of the 53 scenarios; our model allows us to capture the probability of the five categories directly. In addition, we formulate the more general probability of the cases given only that the two connections interact whereas the expressions developed by Garetto et al. are conditioned on a given distance between the sender and receiver.

### 6.1. General approach and preliminaries

We define the interference range and communication range as  $r_i$  and  $r_c$  respectively. The radius of the whole net-

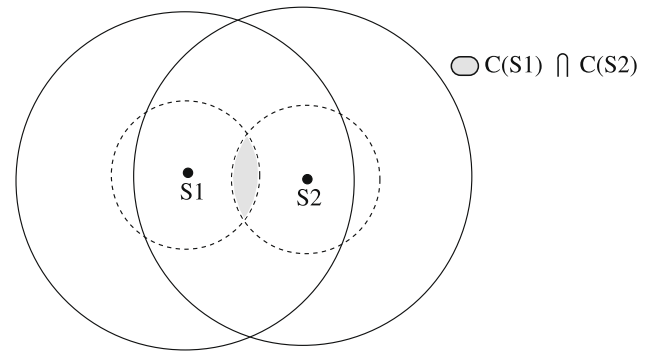
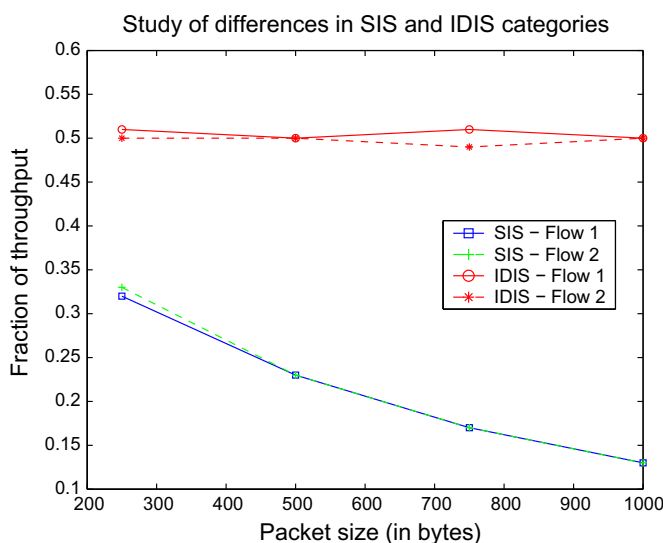


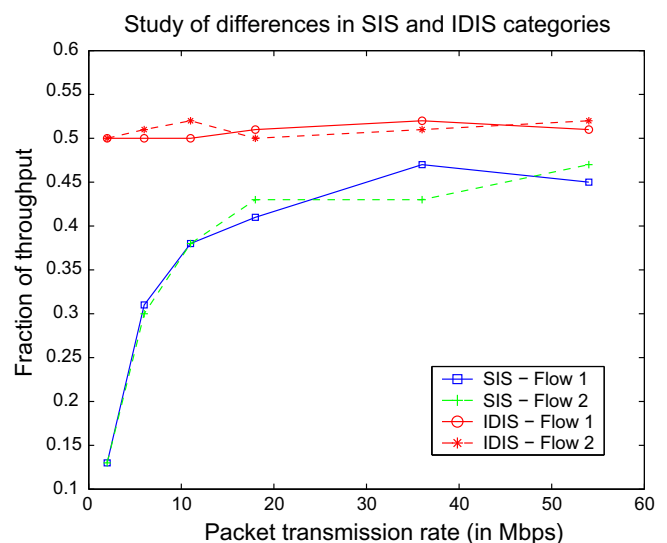
Fig. 8. Intersection of communication range around  $S_1$  and  $S_2$ .

work is represented by  $r_s$ . From the structure of the scenario, since  $D_1$  is the destination of  $S_1$  for one flow these two nodes are always within  $r_c$  of each other, and similarly  $D_2$  is always within  $r_c$  of  $S_2$ . We assume a two disc binary model where a node inside the communication range will receive a message without any errors and a node transmitting from interference range will cause all packets to be dropped at the receiver. We realize that using Signal to Noise ratios SINR is a more accurate measure for determining packet reception and is a topic of our future work. We also assume a uniform distribution of nodes in the network. We use the following terminology:  $C(X)$  refers to the area of communication range of  $X$  (circle of radius  $r_c$  around  $X$ ) and  $T(X)$  refers to the interference range of  $X$  (circle of radius  $r_i$  around  $X$ ). One thing to note is that we have picked  $r_s$  to be  $2r_c + r_i$ , this is the minimum network size to capture all possible scenarios. Increasing the size from here will only increase the percentage of cases with no interactions and hence our evaluation is independent of network size.

In general, the derivation requires computing the area of intersection of two or three circles of different radii.



(a) Throughput vs. Packet Size



(b) Throughput vs. Transmission rate

Fig. 7. Comparison of SIS and IDIS.



While the area of intersection of two circles is well known, computing the area of intersection of three circles is a surprisingly difficult problem. Fortunately, Fewell recently developed expressions for the intersection of three circles, which we apply in our models [20].

For the four secondary channels ( $S_1$ – $S_2$ ,  $D_1$ – $D_2$ ,  $S_1$ – $D_2$  and  $S_2$ – $D_1$ ), the general approach requires computing the probability of the presence (or absence) of a node within  $r_c$  or  $r_i$  from other nodes concurrently as appropriate for the case being modeled. For example, for the SCSl category where  $S_1$  and  $S_2$  are in interference range but the other three secondary links are out of range with each other, this requires computing the area of intersection of  $T(S_1)$  and  $T(S_2)$  that is also outside any of the intersections of  $T(S_1)$  and  $T(D_2)$ , and  $T(S_2)$  and  $T(D_1)$ . This intersection must be computed over all possible distances between  $S_1$  and  $D_1$  and  $S_2$  and  $D_2$ .

## 6.2. SCSl group

For SCSl, the two senders have to be within range of each other (interference or communication). This case occurs if:

- the senders are in interference range or,
- the senders are in communication range and,
  - each sender is in interference range of the other destination or,
  - each sender is in communication range or out of range of the other destination.

To find the probability of the occurrence of this scenario first we calculate the probability of two sources being in interference range. We can get this probability by integrating  $\frac{2x}{r_s^2}$  from  $r_c$  to  $r_i$  as given in the following equation.

$$p_{S_1S_2int} = \int_{r_c}^{r_i} \frac{2x}{r_s^2} dx \quad (1)$$

integrating the same equation from 0 to  $r_c$  will give us the probability that the two sources are in communication range.

$$p_{S_1S_2com} = \int_0^{r_c} \frac{2x}{r_s^2} dx. \quad (2)$$

If the two senders are in communication range, the probability that  $D_2$  is in interference range of  $S_1$  is (shaded region in Fig. 9) given by

$$p_1 = \frac{T(S_1) \cap C(S_2)}{C(S_2)}. \quad (3)$$

Hence  $1 - p_1$  is the probability that  $D_2$  is not in interference range of  $S_1$  (in communication range or out of range). The probability that both receiver are in interference range of opposite sender is  $p_1 p_1$  and the probability that both receivers are not in interference range of opposite sender is  $(1 - p_1)(1 - p_1)$ . Hence if the two sources are within communication range of each other we can find the probability that SCSl occurs as  $p_1^2$  plus the probability  $1 - p_1^2$  times the probability that two sources are in communication range. Thus the probability of SCSl is given by:

$$p_{SCSl} = \int_{r_c}^{r_i} \frac{2x}{r_s^2} dx + \int_0^{r_c} (p_1)^2 (1 - p_1)^2 \frac{2x}{r_s^2} dx. \quad (4)$$

## 6.3. SCAI group

The SCAI group consists of scenarios where the two senders are in communication range and one of the destinations is in interference range of the opposite sender. The other destination is either in communication range or out of range. To calculate the probability of this group we first note that Eq. (2) gives us the probability that two senders are in communication range (shown as shaded region in Fig. 8).

The probability that one destination is within interference range of the opposite sender is calculated as the area of intersection of interference range of the opposite sender and the area of communication range of the sender divided by the communication range of the sender as shown by the shaded region in Fig. 10. More precisely,

$$p_1 = \frac{(T(S_1) \cap C(S_2)) - (C(S_1) \cap C(S_2))}{C(S_2)}. \quad (5)$$

Also the probability that the other destination is either in communication range or out of range of the opposite sender is given by  $p_2$  and  $p_3$  respectively,

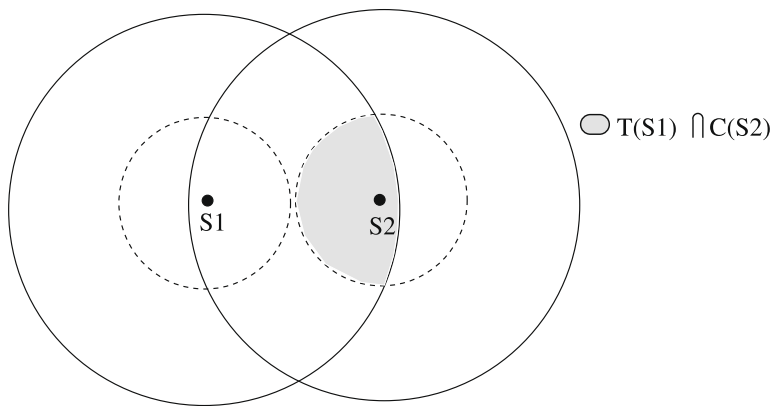
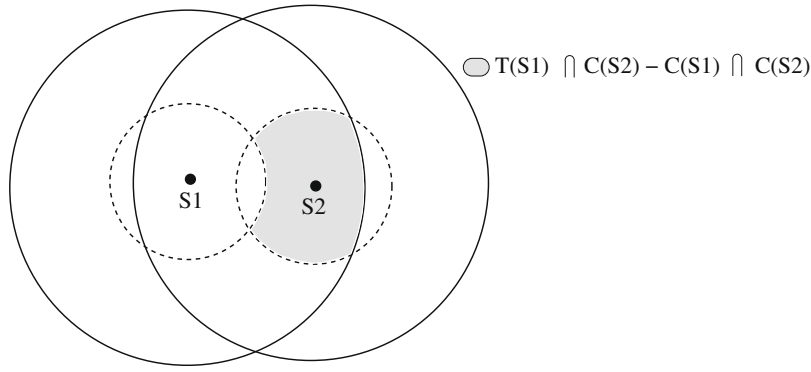


Fig. 9. Intersection of interference range around  $S_1$  and communication range around  $S_2$ .



**Fig. 10.** Shaded region is the area for  $(T(S1) \cap C(S2)) - (C(S1) \cap C(S2))$ .

$$p_2 = \frac{C(S_2) \cap C(S_1)}{C(S_1)}, \quad (6)$$

$$p_3 = \frac{C(S_1) - (T(S_2) \cap C(S_1))}{C(S_1)}. \quad (7)$$

Since there is a symmetric possibility of  $D_2$  being in interference range of  $S_1$ , we need to multiply the total probability by 2.

$$p_{SCAI} = \int_0^{r_c} \frac{4xp_1}{r_s^2} (p_2 + p_3) dx. \quad (8)$$

#### 6.4. AIS group

In AIS, the senders are out of range and one source and the opposite destination are within range (interference or communication range) while the other source and its opposite destination are out of range of each other. To find the probability of this group, we first calculate the probability that the two senders are out of range of each other. Then we find the probability that one destination is within range of the opposite sender. We multiply these two probabilities with the probability that the other destination is out of range of the opposite sender. Eq. (9) gives us the probability that the two senders are out of range.

$$p_1 = \int_{r_i}^{r_s} \frac{2x}{r_s^2} dx. \quad (9)$$

Following equation gives the probability that a destination is out of range of the opposite source

$$p_1 = \frac{C(S_1) - (T(S_2) \cap C(S_1))}{C(S_1)}, \quad (10)$$

while the probability that a destination is within range of opposite source is given by.

$$p_2 = \frac{(T(S_1) \cap C(S_2))}{C(S_2)}. \quad (11)$$

Multiplying these probabilities and adding a factor of 2 (since we can have the reciprocal case also) we get the total probability of AIS group.

$$p_{AIS} = \int_{r_i}^{r_s} \frac{4xp_1p_2}{r_s^2} dx. \quad (12)$$

#### 6.5. SIS group

In SIS group, the senders are out of range and both destinations are in-range (interference or communication) of opposite senders. For the two senders to be out of range at a distance of  $x$  from each other we can use Eq. (16). The probability that destination  $D1$  is at a distance  $y$  from  $S1$  on an arc intersected by circle of radius  $r_i$  around  $S2$  is given by dividing the length of the arc by the area  $C(S1)$ . This would make sure that  $D1$  is in interference range of  $S2$ .

$$p_1 = \frac{2y \cos^{-1}(y^2 + x^2 - r_i^2)}{2xyC(S1)}. \quad (13)$$

Similarly we calculate the probability of  $D2$  being in interference range of  $S1$  while at a distance of  $z$  from  $S2$  as

$$p_2 = \frac{2z \cos^{-1}(z^2 + x^2 - r_i^2)}{2xzC(S1)}. \quad (14)$$

By integrating these two probabilities over  $y$  and  $z$  and multiplying with (16) we get total probability of SIS group as

$$p_{SIS} = \int_{r_i}^{r_s} \int_0^{r_c} \int_0^{r_c} \frac{2x}{r_s^2} p_1 p_2 dx. \quad (15)$$

#### 6.6. IDIS probability derivation

Fig. 11 shows an example placement of nodes for IDIS iteration. For this group, only the destinations are within range of each other (interference or communication) while all other nodes are out of range. We first compute the probability of the two senders being out of range of each other. Then we find the probability that one destination is out of range of the opposite sender. Then we find the probability that the second destination is out of range of its opposite sender as well as the probability that two destinations are in range.

To compute the probability of IDIS we have to calculate (a). The probability that the two sources are out of range of each other (b). The probability that both destinations are out of range of the opposite sources and (c). Given the constraints of (b), the two destinations are within range of each other. To compute (a), the probability that  $S_2$  is at a distance  $x$  from  $S_1$  in a network of radius  $r_s$  is given by  $\frac{2x}{r_s^2}$ , integrating this equation from  $r_i$

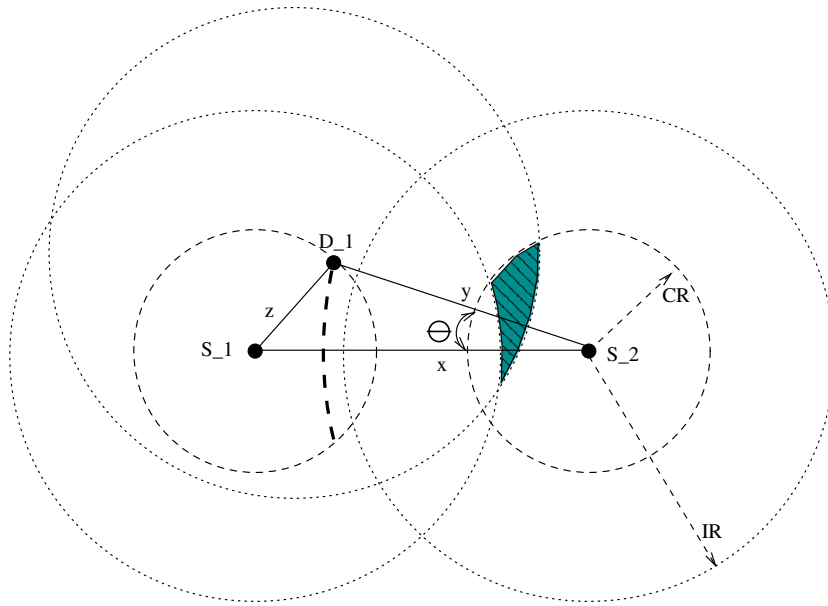


Fig. 11. IDIS group example.

to  $r_s$  will give us the probability  $p_1$  of  $S_2$  being out of range of  $S_1$ . More precisely,

$$p_1 = \int_{r_i}^{r_s} \frac{2x}{r_s^2} dx. \quad (16)$$

We divide (b) in two parts, probability that  $D_1$  is out of range of  $S_2$  and probability that  $D_2$  is out of range of  $S_1$ . First we find the probability of the first part and then we combine the second part with (c). Let us assume that  $D_1$  is at a distance  $y$  from  $S_2$ , we find the probability that  $D_1$  is on an arc at a radius of  $y$  from  $S_2$  which is given by

$$\frac{y dy d\theta}{C(S_1)}, \quad (17)$$

where  $dy$  is the width of the arc and  $\theta$  is the angle  $\angle D_1 S_2 S_1$  as shown in the figure. Because of symmetry we will only consider the values where  $\theta$  is positive and then we multiply by 2 to get the lower half. Since  $D_1$  has to be in communication range of  $S_1$ , the limits of  $\theta$  are from 0 to  $\theta_{\max}$  which is computed as follows.

$$\theta_{\max} = \arccos \frac{x^2 + y^2 - r_c^2}{2xy}. \quad (18)$$

Since we are interested in  $D_1$  being out of range of  $S_2$  the distance  $y$  has a lower limit of  $r_i$ . Its possible for larger values of  $x$ , arc of radius  $y$  around  $S_2$  will not intersect circle of radius  $r_c$  around  $S_1$ , to take care of this case we take the lower limit of  $y$  to be the maximum of  $r_i$  and  $x - r_c$ . The maximum value that  $y$  can take is  $x + r_c$ . Integrating Eq. (17) from 0 to  $\theta_{\max}$  with respect to  $\theta$  and from  $r_i$  to  $x + r_c$  with respect to  $y$  will give us the probability of  $D_1$  being out of range of  $S_2$

$$p_2 = \int_{\max(r_i, x-r_c)}^{x+r_c} \int_0^{\theta_{\max}} \frac{y d\theta dy}{C(S_1)}. \quad (19)$$

To find the probability of  $D_1$  and  $D_2$  being in range we find the area of intersection ( $A(S_2 \cap D_1)$ ) of the circle with radius  $r_i$  around  $D_1$  and the area of the circle with radius  $r_c$  centered at  $S_2$ . Dividing this area by  $C(S_2)$  will give us the probability that  $D_1$  and  $D_2$  are within range. This probability will include those cases where  $S_1$  and  $D_2$  are within range. To remove these cases we subtract from ( $A(S_2 \cap D_1)$ ) the area of intersection of circles of radii  $r_i$  around  $S_1$ ,  $r_i$  around  $D_1$ , and  $r_c$  around  $S_2$ .

$$p_3 = \frac{(C(S_2) \cap T(D_1)) - C(S_2) \cap T(S_1) \cap T(D_1)}{C(S_2)}. \quad (20)$$

The area of intersection of three circles Eq. (16) in [20] requires that the distances between the center of the circles and their radii are known. The distance between  $S_1$  and  $D_1$  is the only unknown in our case which can be calculated by using  $x$  and  $y$  and the angle  $\theta$  between this two line by using the law of cosines

$$z^2 = x^2 + y^2 - 2xy \cos \theta. \quad (21)$$

Combining Eqs. (16), (19), and (20) we get the overall probability of IDIS group by:

$$P(IDIS) = \int_{r_i}^{r_s} \int_{\max(r_i, x-r_c)}^{x+r_c} \int_0^{\theta_{\max}} p_3 \frac{2xy}{r_s^2 C(S_1)} d\theta dy dx. \quad (22)$$

## 6.7. Validation of the geometric models

We validate the geometric models that were developed for the five categories against exhaustive enumeration of the cases. Specifically,  $S_1$  is first placed at a fixed location.  $D_1$  is then moved around  $S_1$  in the entire area of a circular disc with radius equal to the communications range. For every placement of  $S_1$  and  $D_1$ , we move  $S_2$  around  $S_1$  in an area of circular disc of radius  $(r_i + 2r_c)$ . For each location

of  $S_2$ , we place  $D_2$  in the circular area of radius  $r_c$  around  $S_2$ . Note that the above procedure encompasses all the possible legal locations of  $S_2$  and  $D_2$  such that at least one of them interacts with one of  $S_1$  or  $D_1$ . However, it also results in some cases with no interaction which are not of interest to us. We remove those cases from the total count of legal cases to compute the probability. For each of the scenarios we evaluate the interaction between each link to produce the total number of times each scenario will occur.

Fig. 12 plots the occurrence probability of the different groups. We make the following observations:

- The closed form analysis matches closely the results obtained via exhaustive enumeration at all interference ranges.
- If the interference range is equal to the communication range (the case previously studied in [3]), the probability of SCSi increases while the probability of SCAi decreases to 0. As we increase the interference range, while keeping the senders connected, a higher percentage of the area of interference of one source overlaps the area of communication of the other source. This allows for a higher percentage of asymmetrically connected destinations, hence increases SCAi and decreases SCSi.
- As the interference range grows the likelihood of the interference range of a source overlapping with the communication range of the other decreases and hence the probability of SCAi decreases. The same effect also contributes to a decrease in the probability of IDIS.
- The AIS group occurrence decreases as the interference range increases because the probability of a destination being in the communication range of the opposite

source stays the same while the probability that it is in the interference range increases. Hence the probability that both destinations are in interference range of the opposite source increases, contributing more towards the SCSi group and taking away from AIS group.

- The SIS group probability remains relatively constant with changing interference range.

## 7. Throughput estimation model

In this section, we propose a model for the computation of throughput for the proposed categories. We derive the throughput model under a homogeneous network where all the nodes have the same MAC parameters. The channel capacity is denoted by  $C$ . The minimum and maximum backoff window is represented by  $CW_{\min}$  and  $CW_{\max}$ , respectively. The packet loss probability given that the link transmitted a packet (conditional collision probability [21]) is represented by  $p$ . The probability that a source node starts transmission during an idle slot is denoted by  $\tau$ . Bianchi [21] derived the expression for  $\tau$  under Binary Exponential Backoff (BEB) as a function of  $p$  (Eq. (23)).

$$\tau = \frac{2q(1 - p^{m+1})}{q(1 - p^{m+1}) + CW_{\min}[1 - p - p(2p)^{m'}(1 + p^{m-m'}q)]}, \quad (23)$$

where  $q = 1 - 2p$ ,  $m$  is maximum number of retries and  $m'$  is the number of stages to reach  $CW_{\max}$  ( $m' \leq m$ ). The  $p$  and  $\tau$  for the link  $i$  is denoted by  $p_i$  and  $\tau_i$  respectively. Bianchi's

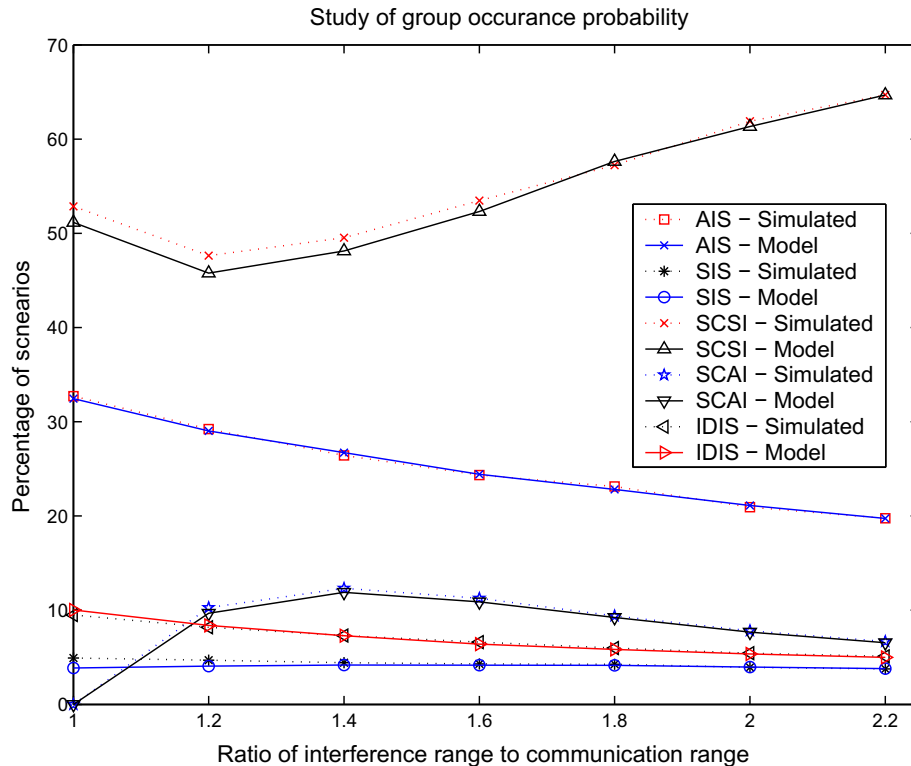


Fig. 12. Occurrence probability of the groups.

model expresses the probability of transmission in a given slot based on the binary exponential backoff model.

We make the following assumptions: (1) The traffic on both the links is saturated. Under less than saturated assumptions, the interactions will play a less important role. and (2) The nodes use the basic mode of IEEE 802.11 (without RTS/CTS), which is becoming the default mode in the network cards due to its superior performance in a majority of the scenarios. Extension of the model by relaxing the above assumptions is an area of future work.

For the SCSI, where hidden terminals do not exist, the throughput can be directly estimated using techniques similar to single-hop wireless network (for example, Bianchi's model [21]). We briefly show the derivation of the model for the four other categories.

### 7.1. General hidden terminal scenario

In this section, we derive a generic model to compute the long-term throughput of the links under hidden terminals using *Renewal Reward Process*. We then specialize this model to account for the different interaction cases. Fig. 13 shows the abstraction of the events observed at a source between two successful packet transmissions.

#### 7.1.1. Modeling long-term throughput as a Renewal Reward process

Consider the process of a source transmitting a packets. Let  $t_s$  and  $t_u$  represent the constant packet transmission durations for a successful and unsuccessful attempt, respectively. The source waits when the channel is idle to decrement its backoff and then transmits the packet. The probability that a link starts transmitting at an idle slot is denoted by  $\tau$  (conditional transmission probability) [21]. The packet may be successfully transmitted or may lead to a collision. Let  $p$  represent the packet loss probability given that the link transmitted a packet (conditional collision probability [21]). Let  $W_i$  be the random variable denoting the wait times before the source transmits the packet. Let  $U_j$  be the number of attempts before successfully transmitting a packet. We assume the following (1)  $W_i$  are iid (independent and identically distributed) random variables; (2) The transmission initiation (transmit or not transmit in a given timeslot) and its result (success or collision) are Bernoulli trials; (3)  $W$  and  $U$  are independent. Under these assumptions, the long-term expected value of  $W$  is given by Eq. (24). The expected value of  $U$  is given by Eq. (25).

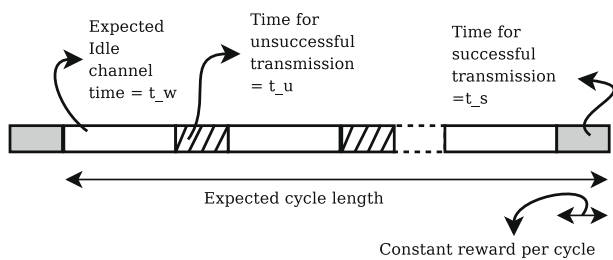


Fig. 13. Packet transmission attempts.

$$E[W] = \frac{1}{\tau}, \quad (24)$$

$$E[U] = \frac{1}{1-p}. \quad (25)$$

Consider the process where a source waits for  $W_i$  and transmits a packet. Consider a renewal process where each cycle ending with a successful transmission. Fig. 13 shows one such cycle. Let  $W_1, W_2, \dots, W_i$  denote the wait times before each transmission and let  $U_j = u$  represent the number of attempts before a successful transmission in one such cycle. We now find the expected value of the time required to complete one cycle (one successful transmission) given that  $U_j = u$ . We note that  $U_j \geq 1$ .

$$\begin{aligned} E[\text{cycle length} | U_j = u] &= E\left[\left(\sum_{i=1}^u (W_i + t_u)\right) - t_u + t_s\right] \\ &= \left(\sum_{i=1}^u (E[W_i] + t_u)\right) - t_u + t_s \\ &= u(E[W_i] + t_u) - t_u + t_s. \end{aligned}$$

The expected value of the cycle length is given by:

$$\begin{aligned} E[\text{cycle length}] &= E[E[\text{cycle length} | U_j = u]] \\ &= E[u](E[W_i] + t_u) - t_u \\ &\quad + t_s (\text{since } W \text{ and } U \text{ are independent}). \end{aligned}$$

We now apply the renewal-reward theory to predict the long-term throughput. The expected reward per cycle is the number of payload bits transmitted in one cycle which is equal to  $Ct_s$ . Hence, the long-term throughput is given by Eq. (27).

$$T_i = \frac{\text{Expected reward per cycle}}{E[\text{cycle length}]} \quad (26)$$

$$= \frac{Ct_s}{n_u(t_w + t_u) - t_u + t_s}, \quad (27)$$

where  $n_u = E[U]$  and  $t_w = E[W]$  as given by Eqs. (24) and (25). The variables that need to be computed are  $p$  and  $\tau$ , which vary based on the type of hidden terminal.

The time required to transmit the DATA packet for link  $i$  is represented as  $l_i$ . We assume that both the links have the same data packet size.

### 7.2. AIS formulation

Recall that in AIS, a source of one link can cause collision at the destination of the other, but not vice versa. Fig. 14 shows this case where a transmission from  $S_1$  can cause a collision at  $D_2$ . The transmission of  $S_2D_2$  will succeed only if it occurs completely during an idle period of the link  $S_1D_1$ . This can lead to severe long term unfairness for  $S_2D_2$ .

We explain the derivation with respect to the scenario in Fig. 14. The throughput estimate for link  $S_1D_1$  is straight-forward since the link does not experience any hidden terminals. Hence, the value of  $p_1 = 0$  and the value of  $\tau_1 = \frac{2}{CW_{\min}}$ .

Let  $p_2$  and  $\tau_2$  represent the  $p$  and  $\tau$  for link  $S_2$ . The link  $S_2D_2$  can transmit only when  $S_1D_1$  is not active, otherwise



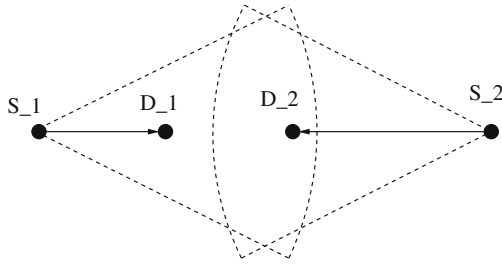


Fig. 14. Hidden terminals in AIS.

the transmission from  $S_1$  will cause a packet collision at  $D_2$ . Let  $p_2$  be the probability that the packet transmitted by  $S_2$  will result in a collision conditioned on  $S_2$  transmitting a packet. Let  $\tau_2$  be the probability that link  $S_2$  starts transmitting at an idle slot.

### 7.2.1. Deriving $p_2$

We first derive the success probability  $(1 - p_2)$  of link  $S_2D_2$ . The packet transmission of  $S_2D_2$  is successful only if the complete packet of  $S_2D_2$  is transmitted when  $S_1$  is inactive. A single slot of overlap between  $S_1D_1$  and  $S_2D_2$  can cause a packet collision at  $D_1$ . Let  $i$  be the congestion window (CW) chosen by the link  $S_1D_1$ . The link  $S_2D_2$  can be successful only if the complete transaction of  $S_1D_1$  lies within that duration of  $i$  slots. For example, as shown in Fig. 15, let  $S_1D_1$  choose a backoff of  $i = 10$  slots and let  $l_2 = 7$  slots. A transmission of  $S_2D_2$  will succeed only if the seven slots of transmission lie within the 10 slots when  $S_1D_1$  is idle. As seen from the Fig. 15, there are four possible arrangements of a successful transmissions out of 10 possible ways.

Generalizing this arrangement of  $l_2$  packet slots in  $i$  slots of idle period, it can be shown that there are  $(i - l_2 + 1)$  ways of placing a successful transmission from  $S_2D_2$ . Let  $p'_2(i)$  be the probability that the transmission from link  $S_2D_2$  succeeds given that  $S_1D_1$  has chosen a backoff window of  $i$ . Eq. (28) gives  $p'_2(i)$  based on the number of successful arrangements of the transmission.

$$\bar{p}_2(i) = \begin{cases} 0 & \text{if } i < l_2, \\ \frac{(i-l_2+1)}{i} & \text{otherwise.} \end{cases} \quad (28)$$

Since the probability of choosing  $i$  from  $[0, CW_{\min_1}]$  is  $\frac{1}{CW_{\min_1} + 1}$  it can be shown that:

$$p_2 = 1 - \frac{\sum_{i=0}^{CW_{\min_1}} \bar{p}_2(i)}{CW_{\min_1} + 1}. \quad (29)$$

Under the BEB scheme, the value of  $\tau_2$  can be calculated by Eq. (23). We also model AIS throughput under a simple scheme where backoff window is always chosen from 0 to  $CW_{\min}$  irrespective of the collision of the transmitted packet (which we refer henceforth as *No backoff* mechanism),  $\tau = \frac{CW_{\min}}{2}$ . The comparison of BEB model with this model helps to identify the effectiveness of BEB. Since we have developed expressions for  $\tau$  and  $p$  for each link in AIS interaction, we can apply Eqs. (27), (24), and (25) from the reward renewal process to calculate the throughput for each link.

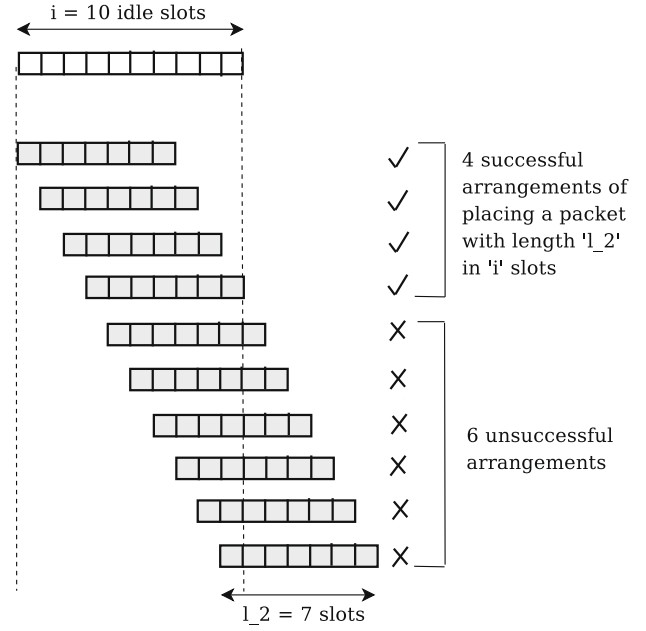


Fig. 15. Packet success in AIS.  $i$  represents the CW chosen by  $S_1D_1$  and  $l_2$  denotes the packet length of the link  $S_2D_2$ .

We now validate the AIS throughput formulation. Under standard MAC parameters (with  $CW_{\min} = 31$ ), the link with the hidden terminal cannot successfully transmit (even relatively smaller packets) between the idle time of the other link (because  $CW_{\min}$  is only 31 slots). Hence, we vary the  $CW_{\min}$  of the links and validate the model for varying  $CW_{\min}$  and packet sizes.

As seen in Fig. 16b, the prediction by the model matches closely with the simulations. It can be seen that the starving link  $S_2D_2$  receives fair throughput only when  $CW_{\min}$  is very high. Exponential backoff at  $S_2$  will reduce the frequency of transmission of link  $S_2D_2$ . However, the interfering traffic at  $S_1D_1$  is at a constant rate and exponential backoff does not improve the success probability of  $S_2D_2$ . The absence of correlation between the change of interference pattern over time makes BEB ineffective in AIS. It can be seen that the link  $S_2D_2$  will get zero throughput unless the  $CW_{\min}$  (of  $S_1D_1$ ) value is large enough to accommodate the the packet. This suggests that under low  $CW_{\min}$ , the effect of asymmetric hidden terminals can be reduced by either decreasing the packet size (or increasing the transmission rate). These parameters can be calculated directly from the model.

### 7.3. SCAI formulation

Under the SCAI category, the sources are within interference range of each other and hence transmissions from the sources do not overlap due to carrier sense. However, the EIFS effect causes one of the links (which we refer to as the 'weaker link') to wait for longer times before decrementing the backoff counter, thus causing throughput degradation.

Let  $\tau_1(\tau_2)$  be the probability that the source of the weaker (stronger) link transmits a packet, conditioned on the channel being idle. Since the links share a common

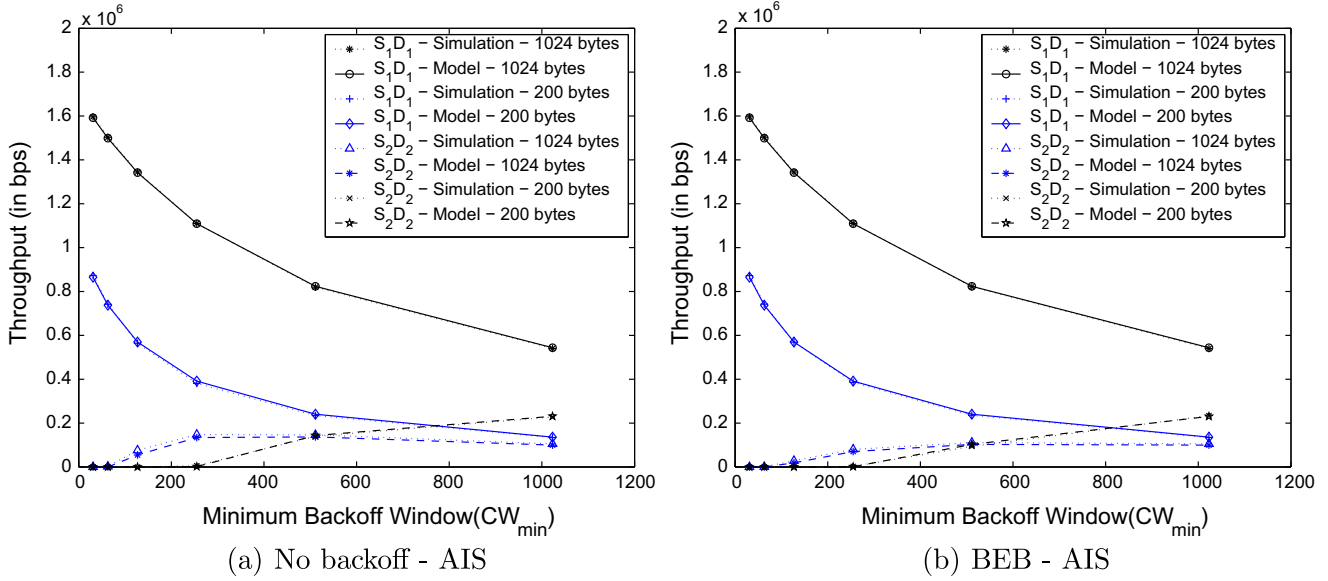


Fig. 16. Effect of hidden terminals.

channel, the probability of winning the channel for transmission by the weaker and the stronger link are in the ratio  $\tau_1 : \tau_2$ . Both the links suffer no hidden terminals ( $p = 0$  for both links). Hence, the throughput of the link  $i$  is given by the Eq. (30).

$$T_i = C \frac{\tau_i}{\tau_1 + \tau_2} \cdot \frac{l_i}{l_i + o_i}, \quad (30)$$

where  $l_i$  denotes the time taken to transmit the data payload and  $o_i$  denotes the average overhead time for transmitting one packet. The overhead time includes the time spent for transmitting header and average backoff period per packet. Hence, the second part of Eq. (30) denotes the fraction of the transmission time that is used to send the payload. The stronger link always transmits with the same probability when the channel is idle. Hence,  $\tau_2$  can be calculated by Eq. (23). The only variable to be computed is  $\tau_1$  to determine the throughput of both the links.

Since we are interested in calculating the *transmission probability conditioned on the channel being idle*, we ignore the time during which the channel is busy. An idle slot can be in one of the backoff/EIFS states (a countable state space). And, the weak link will transmit when the backoff counter is zero (a subset of the state space). Hence, we use a discrete time Markov chain to calculate the probability of transmission at an idle slot  $\tau_1$  shown in Fig. 17 and explained below.

We refer to the source of the weaker link as *the node* in this derivation for clarity purpose. Under an idle slot, the node may be decrementing its backoff or experiencing an EIFS wait period. We also assume that the DIFS period (which is around  $50 \mu s$ ) is zero since it is significantly shorter than the EIFS period (around  $380 \mu s$ ).

In order to compute the state space, we observe that the source may be decrementing its backoff or experiencing an EIFS wait period during an idle slot. The  $i$ th backoff stage is denoted by  $B(i)$  where  $0 \leq i \leq CW_{min}$ . We represent the EIFS duration by  $M$  slots where slot  $j$  represents the num-

ber of slots left for completion of the EIFS duration.  $E(i, j)$  denotes the  $j$ th EIFS slot during the  $i$ th backoff stage and  $0 \leq j \leq M \cdot B(i)$  and  $E(i, j)$  are the states of the chain. The chain is represented in Fig. 17. In this figure, the variable  $\tau_2$  is represented as  $t$ .

The channel becomes busy for the weaker link when the stronger link starts transmitting during an idle slot ( $\tau_2$ ). The value of  $\tau_1$  is dependent upon  $\tau_2$  since the weaker link experiences greater EIFS related backoffs when  $\tau_2$  is higher. However,  $\tau_2$  is independent of  $\tau_1$ . The transition probabilities between the states are represented in Table 1 and are calculated based on the following set of rules.

During the backoff period, a node will move from backoff stage  $B(i)$  to backoff state  $B(i-1)$  when the channel is sensed idle at the end of a slot (Rule 1). If the channel is sensed busy, it will freeze the backoff and start its EIFS (at state  $E(i, M)$ ) once the channel becomes idle again (Rule 2). While in EIFS, the node will decrement the number of EIFS slots to wait if the channel is sensed idle (Rule 3). If the channel becomes busy during an EIFS, the node will resume EIFS from the start when the channel is sensed idle again, hence moving to the state  $E(i, M)$  (Rule 4). When the backoff stage reaches 0 (stage  $B(0)$ ), the node will transmit the packet and then choose a uniform random backoff from  $[0, CW_{min}]$  upon the successful completion of DIFS (Rule 6). Similar explanations can be provided for the other rules.

The node starts transmitting the packet only when the channel is idle at the slot boundary when: (1) the backoff counter is zero (state  $B(0)$ ); or (2) The EIFS period is completed and backoff counter is zero (state  $EIFS(0, 0)$ ). Hence, the probability with which the node starts transmitting a packet at an idle time slot ( $\tau_1$ ) is given by Eq. (31).

$$\tau_1 = (1 - \tau_2)(\Pi_{B(0)} + \Pi_{EIFS(0,0)}), \quad (31)$$

where  $\Pi$  are the limiting probabilities of the above chain.

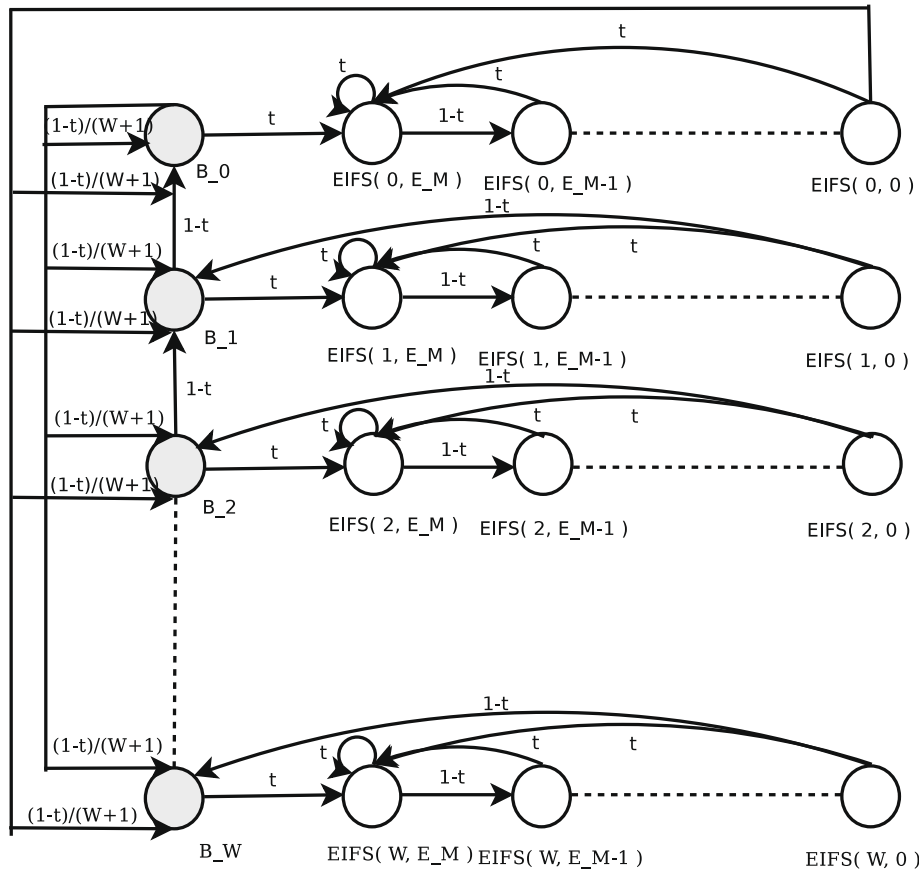


Fig. 17. Markov chain for EIFS calculation.

**Table 1**  
Transition probabilities.

Rule	From	To	Probability
1	$B(i), i \neq 0$	$B(i-1)$	$1 - \tau_2$
2	$B(i)$	$E(i, M)$	$\tau_2$
3	$E(i, j), j \neq 0$	$E(i, j-1)$	$1 - \tau_2$
4	$E(i, j)$	$E(i, M)$	$\tau_2$
5	$E(i, 0), i \neq 0$	$B(i)$	$1 - \tau_2$
6	$B(0)$	$B(i)$	$\frac{1 - \tau_2}{CW_{min} + 1}$
7	$E(0, 0)$	$B(i)$	$\frac{1 - \tau_2}{CW_{min} + 1}$

Fig. 18 validates the model by comparing it with simulation (with standard MAC parameters). The simulation was conducted using the QualNet simulator, which implements a detailed model of IEEE802.11 [22]. Packet size was varied from 200 bytes to 1024 bytes. Since the links compete with a ratio  $\tau_1 : \tau_2$ , a constant ratio of the throughput between the weak and the strong link can also be seen (24% according to the simulations and 32% according to the model). The comparison between the model and the simulation indicates that the assumptions of the model (especially the discrete EIFS slots and independence of  $\tau_1$ ) are reasonable.

We now model the categories with symmetric hidden terminals (SIS and IDIS).

The models include the impact of the backoff mechanisms in a manner that allows different strategies to be evaluated. We first derive a general throughput estimation

model and discuss the effect of hidden terminals with respect to this model.

#### 7.4. Preliminary formulation for symmetric categories

Symmetric hidden terminals occurs in the SIS and IDIS categories. Computation of the throughput variables  $p$  and  $\tau$  for symmetric categories is hard due to the coupling between the two links. This makes independence assumptions on the probability of collision per backoff stage inaccurate. An accurate model of these cases would require modeling the combined states of the two senders (each of which may take any of the states in the Bianchi model), leading to a very large Markov chain. Nevertheless, we present results with the approximate model. We believe that an accurate model of SIS cases is an open question that deserves a more thorough treatment.

##### 7.4.1. SIS formulation

In SIS, both sources ( $S_1$  and  $S_2$ ) cause a collision at the destination of the other link ( $D_2$  and  $D_1$ ). Reception at  $D_2$  is successful if  $S_1$  does not transmit in slots that overlap with  $S_2$ 's transmission. Due to symmetry of the hidden terminals, we have  $p = p_1 = p_2$  and  $\tau = \tau_1 = \tau_2$ .

We now derive the the success probability  $(1 - p)$ . If the value of maximum backoff is lesser than the time required to complete a successful transmission (and ACK), then  $S_2D_2$

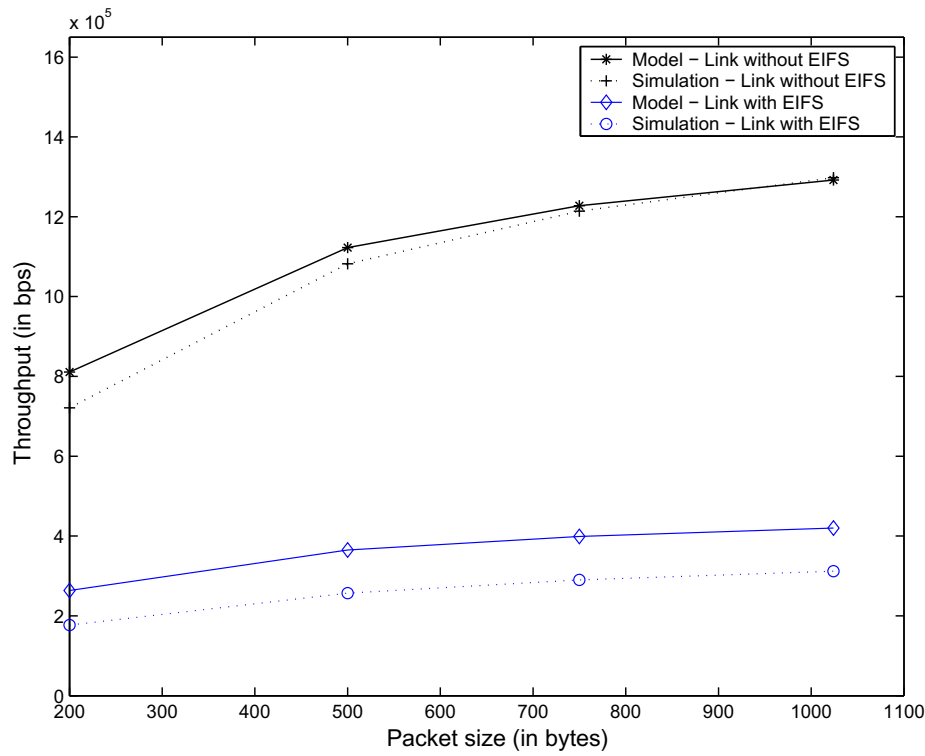


Fig. 18. Throughput study for EIFS effect.

cannot find a sufficiently long gap between  $S_1D_1$ 's transmission and hence  $p = 0$ . Otherwise, the packet will be transmitted successfully if the interfering link does not transmit a packet such that at least one slot overlaps with the packet transmission. For example, Fig. 19 shows the transmission of the data packet by one link (say  $S_1D_1$ ) in light blue. The slots during which interfering link ( $S_2D_2$ ) cannot transmit is colored in grey. If  $\tau$  is the probability that the interfering link will transmit in a given time slot, then the probability that it will transmit in the slots that will collide with the given packet is given by

$$p = \tau + (1 - \tau)\tau + \dots + (1 - \tau)^{[2\ell]-1}\tau, \quad (32)$$

$$p = 1 - (1 - \tau)^{2[\ell]}.$$

The relationship between  $p$  and  $\tau$  is given by the Eq. (23) which we term  $\tau = b(p)$ .

Symmetric hidden terminals have the property of one link being affected by the activity of the other. The probability of drop on link  $S_1D_1$  ( $p_1$ ) depends on the frequency of packet transmission attempts at link  $S_2D_2$  ( $\tau_2$ ). Let us denote this relation by  $p_1 = f(\tau_2)$ . Owing to symmetry in the topology, we can represent the above by two relations: (1)  $\tau = b(p)$ ; and (2)  $p = f(\tau)$ . Eq. (32) can be used as an approximation for representing the function  $p = f(\tau)$ . The roots of the above equations can be calculated by standard numerical techniques like Newton method [23]. Improving the expression for  $f$  from that in Eq. (32) is a part of our future work. Integrating the existing components for calculating  $p$  and  $\tau$  was also attempted. A Markov chain based approach to calculate  $\tau$  was proposed by Garetto et al. [3]. Fig. 20b study the throughput of the links when the  $CW_{min}$  and packet sizes are altered. It compares the simulation with the two models (the Garetto model and the one proposed here). Our model matches the simulation only under higher  $CW_{min}$  values. The Markov-chain based model captures the throughput trend for larger packet sizes, while a large gap exists under lower packet sizes. The model is accurate for No-Retry mechanism (Fig. 16a). At lower values of  $CW_{min}$ , the BEB scheme outperforms the No-retry scheme. The exponential backoff of one link helps to create enough channel idle time for packet transmissions of the other link. However, such a scenario exhibits short term unfairness where the throughput of one link dominates for short periods of time.

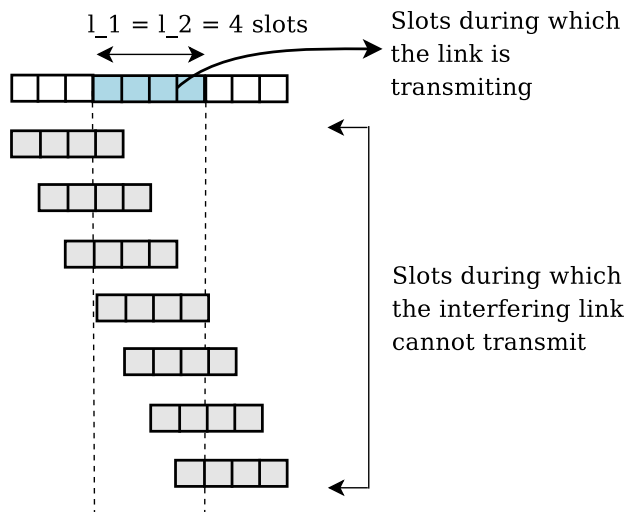


Fig. 19. Packet success in SIS.  $l_1$  and  $l_2$  represents the slots required to transmit the packet for link  $S_1D_1$  and  $S_2D_2$ .

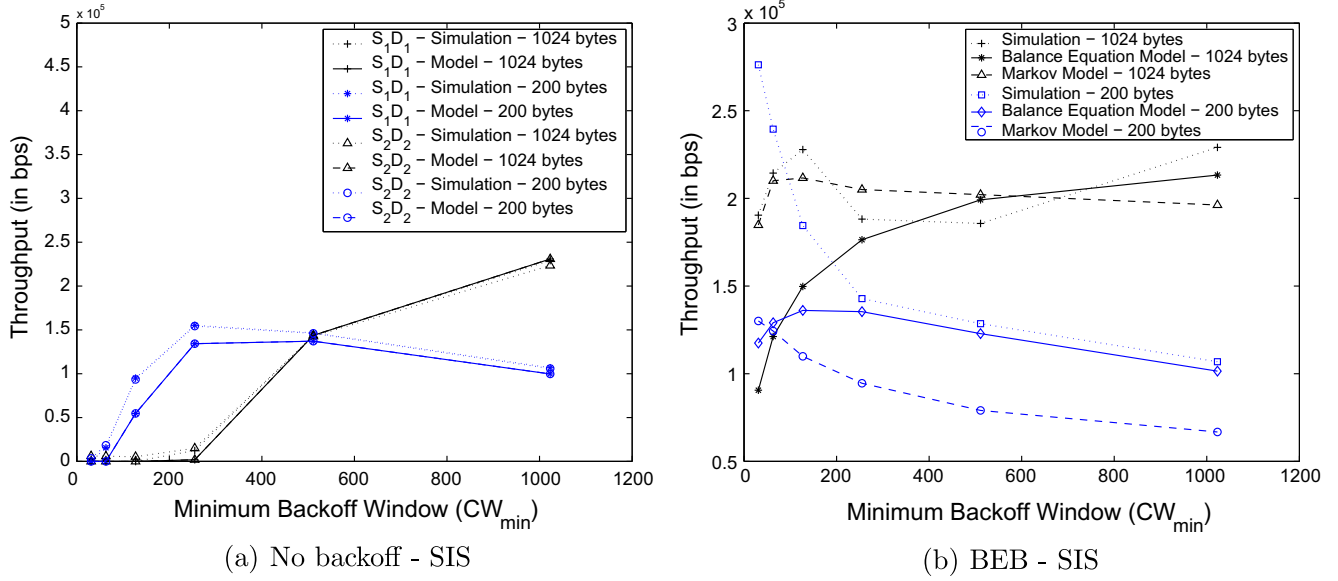


Fig. 20. Effect of hidden terminals.

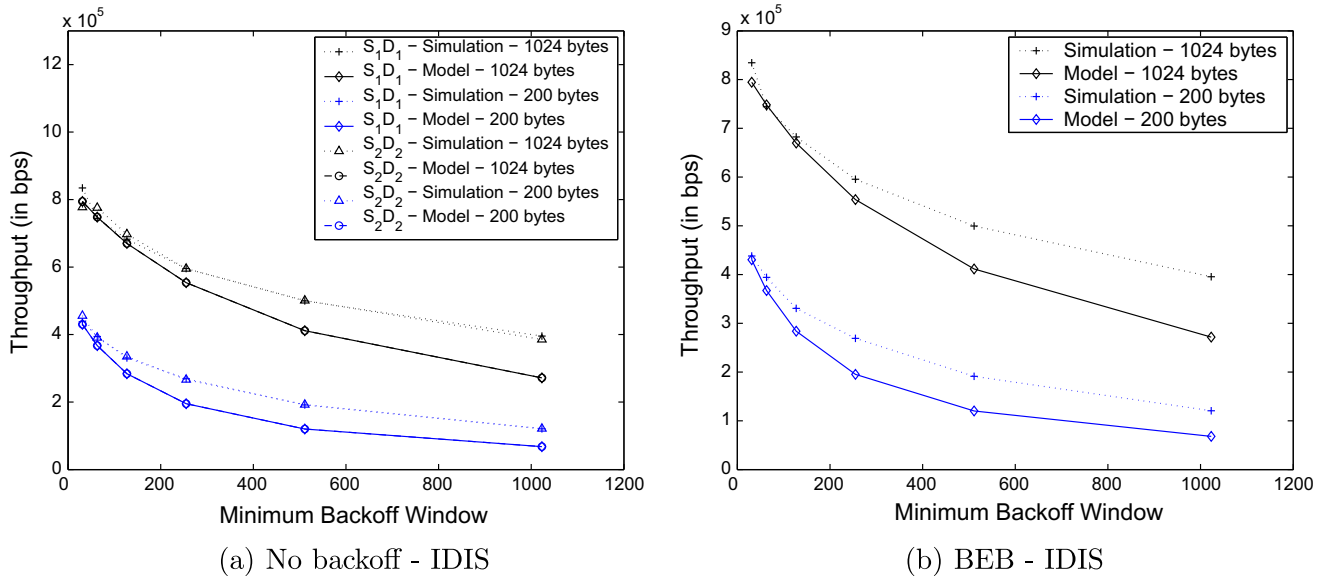


Fig. 21. Effect of hidden terminals.

#### 7.4.2. IDIS formulation

Recall that in IDIS, only receivers are in interference range with each other. A receiver can cause a drop on the other link when it transmits an ACK. Due to the symmetry of the topology,  $p$  and the  $\tau$  are identical for the two links, but are coupled. Their value can be derived in a method similar to symmetric hidden terminals, under the same imperfect assumptions. The results of this model are shown in Fig. 21a and b.

### 8. Concluding remarks

The paper makes several contributions to the analysis of two single hop wireless flows. Specifically, it relaxes the assumption of a constant interference-range (also carrier sense range) to communication-range ratio in existing

two-flow models, which does not hold in practical radios. Additional types of interactions occur under these assumptions, which the paper categorizes. It also develops closed form expressions for the probability of occurrence of the scenarios and analyzes their frequency as a function of the interference/carrier sense range. The paper also contributes constructive models for the throughput in presence of hidden terminals, although the models for SIS remain quite approximate.

Our most immediate future work includes generalizing the model to account for a more realistic physical environment based on Signal to Interference and Noise Ratio. In addition, we plan to study the impact of interference in a chain topology from first principles. We also seek to improve the throughput models we developed for symmetric hidden terminals which do not match simulation in all



cases. Such a model will allow us to account for the effect of capture, and likely expose additional interaction cases more representative of realistic radio operation.

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