- Hidden Surface Removal
  - Back Face Culling
- 3D Surfaces
  - Bicubic Parametric Bezier Surface Patches
- 3D Graphics with OpenGL

Back-Face Culling

- Define one side of each polygon to be the visible side
  - That side is the outward-facing side
- Defining each polygon in the polygons array:
  - Systematically number vertices in counter-clockwise fashion as seen from outside of the object

First: Review of Vector Products

- Dot (Scalar) Product

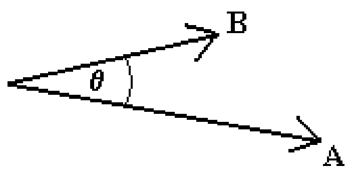
  s = A · B

  s = |A| * |B| * cos(θ)

  θ is the angle between vectors A and B

  In terms of components (RH coord system):

  s = Ax*Bx + Ay*By + Az*Bz



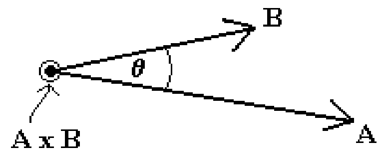
Cross (Vector) Product

- V = A x B, a vector
- Magnitude: |V| = |A| * |B| * sin(θ)

  θ is angle between vectors A and B
- Direction: Given by right-hand rule
  - 1. Align fingers of right hand with first vector
  - 2. Rotate toward second
  - 3. Thumb points in direction of V

In the following diagram:

$V = A \times B$ would point out of the screen toward the observer



In terms of components (RH coordinate system):

$$V = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad \text{(a determinant)}$$

i, j, k are unit vectors along x,y,z axes

Triple Product

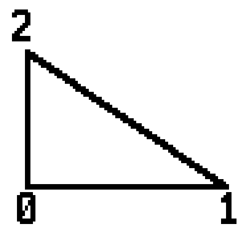
$A \cdot (B \times C)$

$$A \cdot (B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \qquad \text{(determinant)}$$

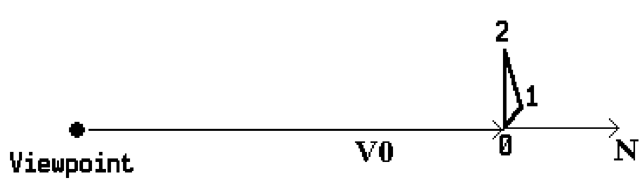
(Components in terms of RH coord system)

Back-Face Culling

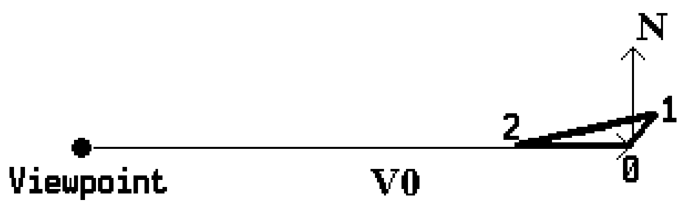
- Consider triangle with vertices 0, 1, 2
- Visible side of the triangle: 0,1,2
  - Vertices numbered in counter-clockwise order
  - Invisible side is: 0,2,1
    - (clockwise vertex ordering)



- Define vector N
  - Outward normal to triangle
- Define Vector V0
  - Vector from observer to vertex 0
- Some Cases:
  - N and V0 nearly parallel (V0 · N = 1)
  - Visible side of triangle 0 1 2 invisible to viewer



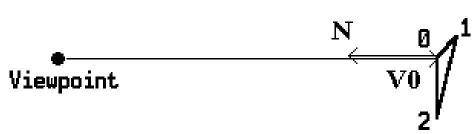
- Rotate triangle about side 01 by 90 degrees
  - Now N and V0 are perpendicular (V0 · N = 0)
  - Triangle is about to become visible
  - At all other points between these two orientations:
    - V0 · N is positive
    - And triangle is invisible to viewer



- Continue rotation about side 01
- Triangle becomes visible to the viewer
- 90 degrees more, N and V0 are antiparallel

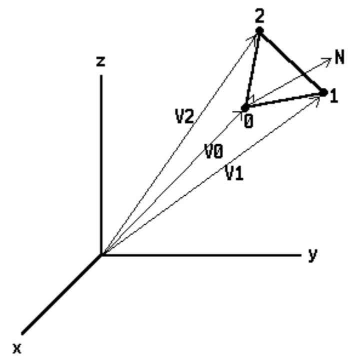
  V0 · N = -1

  Triangle facing toward viewer and is visible
  - At all intermediate orientations:
    - Triangle is visible
    - And V0 · N is negative



Criterion for Invisibility

- If $V0 \cdot N > 0$, triangle 012 is invisible
- Now place triangle 012 in an arbitrary position relative to viewer V



- Outward normal N is vector (cross) product of V01 and V02

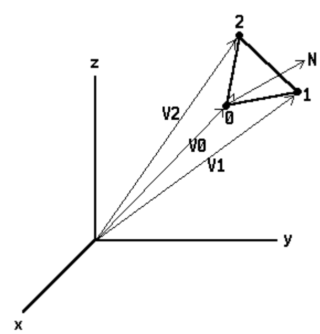
  V01 is vector from vertex 0 to vertex 1

  V02 is vector from vertex 0 to vertex 2
- So: $N = V01 \times V02$
- Criterion for invisibility:

  $V0 \cdot (V01 \times V02) > 0$
- But:

  $V01 = V1 - V0$

  $V02 = V2 - V0$



- Substituting we get:

  $V0 \cdot [(V1 - V0) \times (V2 - V0)] > 0$, invisibility

- Expanding:

  $V0 \cdot (V1 \times V2) - V0 \cdot (V1 \times V0) - V0 \cdot (V0 \times V2) + V0 \cdot (V0 \times V0) > 0$

- Last Term = 0

  (Cross product of any vector with itself = 0)

- Middle two terms:

  Quantity inside ( ) is a vector perpendicular to V0

  So dot product of either vector with V0 is 0

So: $V0 \cdot (V1 \times V2) > 0$

- For right-handed coordinate system, triple product can be expressed as a determinant

$$V0 \cdot (V1 \times V2) = \begin{vmatrix} X0 & Y0 & Z0 \\ X1 & Y1 & Z1 \\ X2 & Y2 & Z2 \end{vmatrix}$$

- (X0,Y0,Z0), (X1,Y1,Z1), (X2,Y2,Z2) are viewing coordinates (xv,yv,zv) of vertices 0, 1, and 2
- But viewing coordinate system is <u>left-handed</u>
- So sign of the determinant must be reversed

Final Criterion for Invisibility

$$\begin{vmatrix} X0 & Y0 & Z0 \\ X1 & Y1 & Z1 \\ X2 & Y2 & Z2 \end{vmatrix} < 0$$

- Result can be applied to any planar polygon
- Use viewing coordinates of three consecutive polygon vertices
- Could implement as a “visibility” function
  - Computes and returns value of determinant
    - Positive means visible, negative invisible

3-D Surfaces

- Explicit Representation

  z = f(x,y)
- Plotting
  - Fix values of y and vary x
  - Gives a family of curves

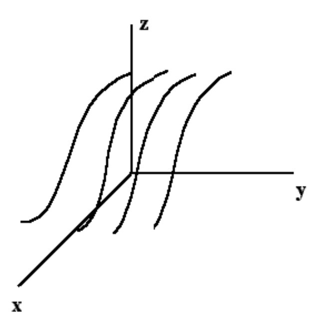
    z0 = f(x,0)

    z1 = f(x,δ)

    z2 = f(x,2*δ)

    z3 = f(x,3*δ)

    etc.



Plotting 3D Surfaces, continued

- Then fix values of x and vary y
- Gives another family of curves

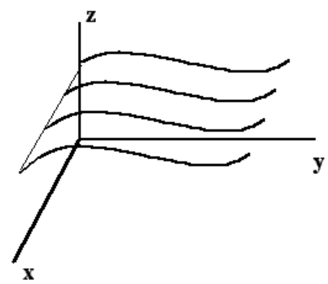
  z0' = f(0,y)

  z1' = f(δ,y)

  z2' = f(2*δ,y)

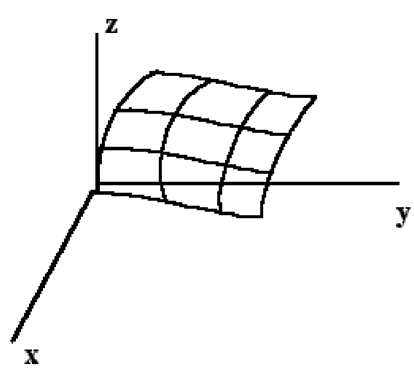
  z3' = f(3*δ,y)

  etc.



Plotting 3D Surfaces, continued

- Result is a wireframe that represents the surface
- Could be broken up into polygons



Parametric Representation of 3D Surfaces

- Need two parameters, say t and s
- $x = x(t,s)$, $y = y(t,s)$, $z = z(t,s)$
- both t and s vary over a range (0 to 1)
- To plot:
  - Fix values of s and for each vary t over range
    - gives one family of isoparametric curves
  - Fix values of t and for each vary s over range
    - gives another family of isoparametric curves

Cubic Bezier Curves (Review)

- In matrix form, points on curve P [P = x,y] are given in terms of paramter t and four control points P0, P1, P2, P3
- Result:

  $P = a*t^3 + b*t^2 + c*t + d, \quad 0 <= t <= 1$
  - Can be written in a more compact form:

    P = T * Bg * Pc

    T: row vector of parameter powers $[\ t^3\ \ t^2\ \ t\ \ 1\ ]$

    Bg: the constant 4 X 4 Bezier Geometry matrix

    Pc: column vector of the control points

Bicubic Bezier Surface Patches

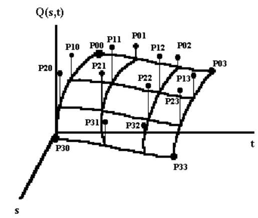
- Define 4-vectors S and T:

$$S = \begin{bmatrix} s^3 & s^2 & s & 1 \end{bmatrix},\ 0 <= s <= 1$$

$$T = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix},\ 0 <= t <= 1$$

- Define points on surface patch Q(s,t) [Q = x,y,z] as:

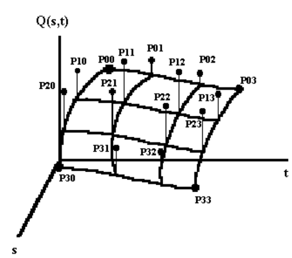
$$Q(s,t) = S * M_B * \begin{bmatrix} P0(t) \\ P1(t) \\ P2(t) \\ P3(t) \end{bmatrix}$$



Control points P0, P1, P2, P3 are themselves parameterized by t

M_B is the Bezier Geometry Matrix we've seen before

$$\text{So } P0(t) = T * M_B * \begin{bmatrix} P00 \\ P01 \\ P02 \\ P03 \end{bmatrix}$$



Transposing:

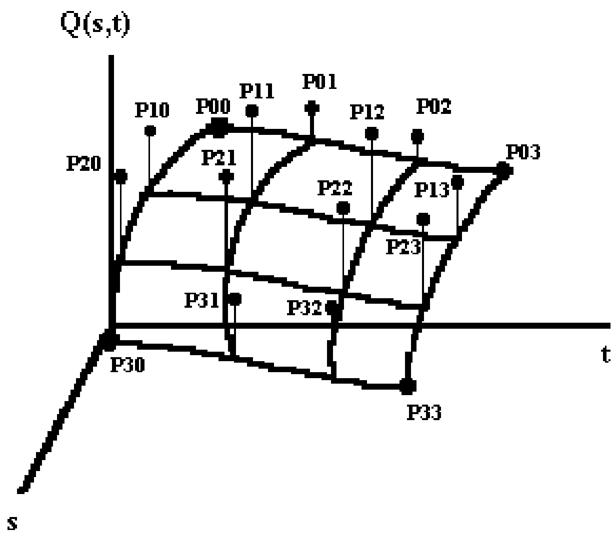
$$P0(t) = \begin{bmatrix} P00 & P01 & P02 & P03 \end{bmatrix} * M_B^T * T^T$$

Do the same for P1(t), P2(t), P3(t)

Result:

$$Q(s,t) = S * M_B * \begin{bmatrix} P00 & P01 & P02 & P03 \\ P10 & P11 & P12 & P13 \\ P20 & P21 & P22 & P23 \\ P30 & P31 & P32 & P33 \end{bmatrix} * M_B^T * T^T$$

A Bicubic Bezier Surface Patch



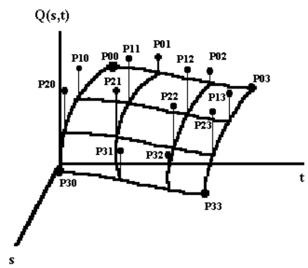
Expanding and Rearranging Terms -- x(s,t) Equation

$$
\begin{aligned}
x(s,t) = {} & (1-s)^3 \left[x_{00}(1-t)^3 + 3x_{01}(1-t)^2 t + 3x_{02}(1-t)t^2 + x_{03}t^3\right] \\
& + 3(1-s)^2 s \left[x_{10}(1-t)^3 + 3x_{11}(1-t)^2 t + 3x_{12}(1-t)t^2 + x_{13}t^3\right] \\
& + 3(1-s)s^2 \left[x_{20}(1-t)^3 + 3x_{21}(1-t)^2 t + 3x_{22}(1-t)t^2 + x_{23}t^3\right] \\
& + s^3 \left[x_{30}(1-t)^3 + 3x_{31}(1-t)^2 t + 3x_{32}(1-t)t^2 + x_{33}t^3\right]
\end{aligned}
$$

- Similar equation for y(s,t)

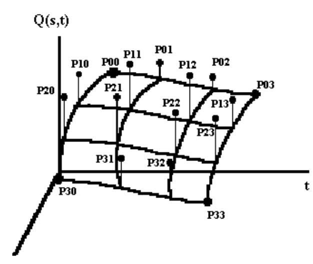
Plotting One Set of Isoparametric Curves

```
For (s=0; s<=1; s+=δ)
    Compute & store x(s,0), y(s,0), z(s,0)
    Project to screen and store --> xs(s,0), ys(s,0)
    MoveTo(xs(s,0), ys(s,0))
    For (t=0; t<=1; t+=δ)
        Compute & store x(s,t), y(s,t), z(s,t)
        Project to screen and store --> xs(s,t), ys(s,t)
        LineTo(xs(s,t), ys(s,t))
```



Plotting the Other Set of Isoparametric Curves

```
For (t=0; t<=1; t+=δ)
    MoveTo(xs(0,t), ys(0,t))
    For (s=0; s<=1; s+=δ)
        LineTo(xs(s,t), ys(s,t))
```



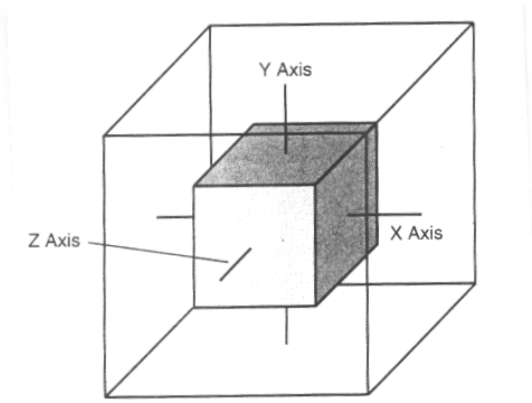
Introduction to 3D Graphics with OpenGL

3D Graphics Using OpenGL

- Building Polygon Models
- ModelView & Projection Transformations
- Quadric Surfaces
- User Interaction
- Hierarchical Modeling
- Animation

OpenGL 3D Coordinate System

- A Right-handed coordinate system
  - Viewpoint is centered at origin initially



Defining 3D Polygons in OpenGL

- e.g., front face of a cube

```
glBegin(GL_POLYGON)
    glVertex3f(-0.5f, 0.5f, 0.5f);
    glVertex3f(-0.5f, -0.5f, 0.5f);
    glVertex3f(0.5f, -0.5f, 0.5f);
    glVertex3f(0.5f, 0.5f, 0.5f);
glEnd();
```

- need to define the other faces

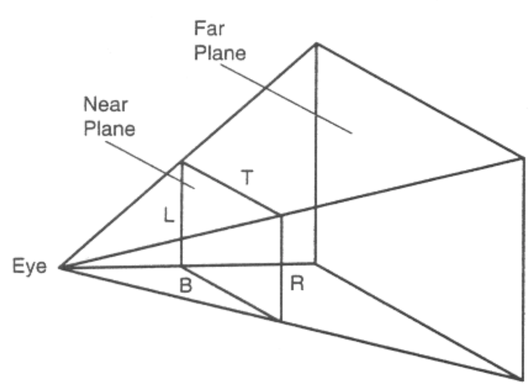
Projection Transformation

- First tell OpenGL you're using the projection matrix
  glMatrixMode(GL_PROJECTION);
- Then Initialize it to the Identity matrix
  glLoadIdentity();
- Then define the viewing volume, for example:
  glFrustum(-1.0, 1.0, -1.0, 1.0, 2.0, 7.0);
  - (left, right, bottom, top, near, far)
    - near & far are positive distances, near < far
  - Viewing volume is the frustum of a pyramid
  - Used for perspective projection

  or glOrtho(-1.0, 1.0,-1.0, 1.0, 2.0, 7.0);
  - Viewing volume is a rectangular solid
  - for parallel projection
- For both the viewpoint (eye) is at (0,0,0)

The Viewing Volume

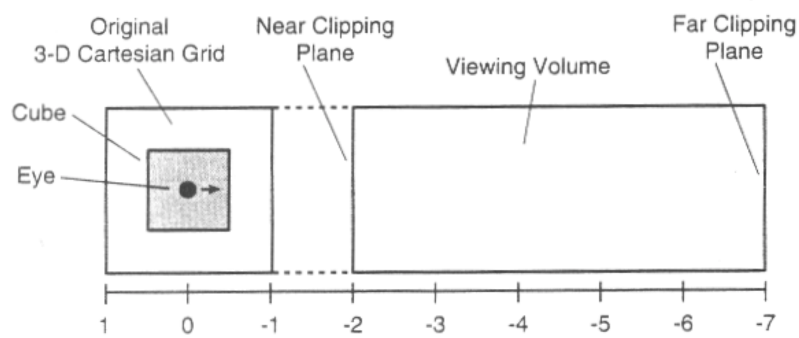
- Everything outside viewing volume is clipped
- Think of near plane as being window's client area



Modelview Transformation

Our cube is not visible

It lies in front of near clipping plane



Positioning the Camera

- By default it's at (0,0,0), pointing in –z direction, up direction is y-axis
- Can set the camera point
- And the "lookat" point
- And the up direction

  gluLookAt(xc,yc,zc,xa,ya,za,xu,yu,zu);

  (xc,yc,zc) coordinates of virtual camera

  (xa,ya,za) coordinates of lookat point

  (xu,yu,zu) up direction vector
- Example:

  gluLookAt(2.0,2.0,2.0,0.0,0.0,0.0,0.0,0.0,1.0);

  camera at (2,2,2), looking at origin, z-axis is up

Modelview Transformation

- Used to perform geometric translations, rotations, scalings
- Also implements the viewing transformation
- If we don't position the camera, we need to move our cube into the viewing volume

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslate(0.0f, 0.0f, -3.5f);
```

  - Translates cube down z-axis by 3.5 units

- OpenGL performs transformations on all vertices
- First modelview transformation
- Then projection transformation
- The two matrices are concatenated
- Resulting matrix multiplies all points in the model

OpenGL Geometric Transformations

- "Modeling" Transformations

```
glScalef(2.0f, 2.0f, 2.0f);  // twice as big
   parameters: sx, sy, sz

glTranslatef(2.0f, 3.5f, 1.8f); // move object
   parameters: tx, ty, tz

glRotatef(30.0f, 0.0f, 0.0f, 1.0f); // 30 degrees about z-axis
   parameters:
    – angle
    – (x,y,z) -> coordinates of vector about which to rotate
```

OpenGL Composite Transformations

- Combine transformation matrices
- Example: Rotate by 45 degrees about a line parallel to the z axis that goes through the point (xf,yf,zf) – the fixed point

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslate(xf,yf,zf);
glRotate(45, 0.0,0.0,1.0);
glTranslate(-xf,-yf,-zf);
```

- <u>Note last transformation specified is first applied</u>
  - Because each transformations in OpenGL is applied to present matrix by postmultiplication

Typical code for a polygon mesh model

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glFrustum(-1.0, 1.0, -1.0, 1.0, 2.0, 7.0);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslatef(0.0f, 0.0f, -3.5f);              // translate into viewing frustum
glRotatef(30.0f, 0.0f, 0.0f, 1.0f);           // rotate about z axis by 30
glClearColor(1.0f, 1.0f, 1.0f, 1.0f);         // set background color
glClear(GL_COLOR_BUFFER_BIT);                 // clear window
glColor3f(0.0f, 0.0f, 0.0f);                  // drawing color
glPolygonMode(GL_FRONT_AND_BACK, GL_LINE);
glBegin(GL_POLYGON);
  //define polygon vertices here
glEnd();
```

- See 3dxform example program

The OpenGL Utility Library (GLU) and Quadric Surfaces

- Provides many modeling features
  - Quadric surfaces
    - described by quadratic equations in x,y,z
    - spheres, cylinders, disks
    - Polygon Tesselation
      - Approximating curved surfaces with polygon facets
  - Non-Uniform Rational B-Spline Curves & Surfaces (NURBS)
- Routines to facilitate setting up matrices for specific viewing orientations & projections

Modeling & Rendering a Quadric with the GLU

1. Get a pointer to a quadric object
2. Make a new quadric object
3. Set the rendering style
4. Draw the object
5. When finished, delete the object

OpenGL GLU Code to Render a Sphere

```
GLUquadricObj *mySphere;
mySphere=gluNewQuadric();
gluQuadricDrawStyle(mySphere,GLU_FILL);
    // some other styles: GLU_POINT, GLU_LINE
gluSphere(mySphere,1.0,12,12);
    // radius, # longitude lines, # latitude lines
```

The GLUT and Quadric Surfaces

- Many predefined quadric surface objects
  - glutWire***()
  - glutSolid***()
  - Some examples:
    - glutWireCube(size); glutSolidCube(size);
    - glutWireSphere(radius,nlongitudes,nlatitudes);
    - glutWireCone(rbase,height,nlongitudes,nlatitudes);
    - glutWireTeapot(size);
    - Lots of others
  - See cone_perspective example program

Interaction in OpenGL

- OpenGL GLUT Callback Functions
  - GLUT's version of event/message handling
  - Programmer specifies function to be called by OS in response to different events
  - Specify the function by using glut***Func(ftn)
    - We've already seen glutDisplayFunc(disp_ftn)
    - disp_ftn called when client area needs to be repainted
      - Like Windows response to WM_PAINT messages
  - All GLUT callback functions work like MFC On***() event handler functions

Some Other GLUT Callbacks

- glutReshapeFunc(ftn(width,height))
  - Identifies function ftn() invoked when user changes size of window
    - height & width of new window returned to ftn()
- glutKeyboardFunc(ftn(key,x,y))
  - Identifies function ftn() invoked when user presses a keyboard key
  - Character code (key) and position of mouse cursor (x,y) returned to ftn()
- glutSpecialFunction(ftn(key,x,y))
  - For special keys such as function & arrow keys

Mouse Callbacks

- glutMouseFunc(ftn(button, state, x, y))
  - Identifies function ftn() called when mouse events occur
    - Button presses or releases
    - Position (x,y) of mouse cursor returned
    - Also the state (GLUT_UP or GLUT_DOWN)
    - Also which button
      - GLUT_LEFT_BUTTON, GLUT_RIGHT_BUTTON, or GLUT_MIDDLE_BUTTON

Mouse Motion

- Move event: when mouse moves with a button pressed –
  - glutMotionFunctionFunc(ftn(x,y))
    - ftn(x,y) called when there's a move event
    - Position (x,y) of mouse cursor returned
- Passive motion event: when mouse moves with no button pressed
  - glutPassiveMotionFunctionFunc(ftn(x,y))
    - ftn(x,y) called when there's a passive motion event
    - Position (x,y) of mouse cursor returned

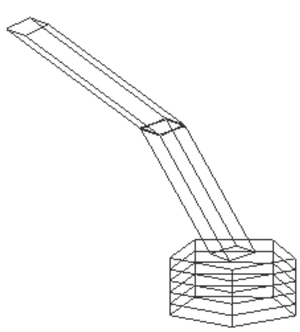
GLUT Menus

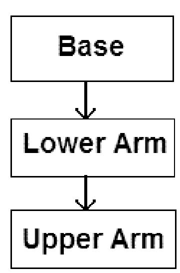
- Can create popup menus and add menu items with:
  - glutCreateMenu (menu-ftn(ID))
    - Menu-ftn(ID) is callback function called when user selects an item from the menu
    - ID identifies which item was chosen
  - glutAddMenuEntry(name, ID_value)
    - Adds an entry with name displayed to current menu
    - ID_value returned to menu_ftn() callback
  - glutAttachMenu(button)
    - Attaches current menu to specified mouse button
    - When that button is pressed, menu pops up

Hierarchical Models

- In many applications the parts of a model depend on each other
- Often the parts are arranged in a hierarchy
  - Represent as a tree data structure
  - Transformations applied to parts in parent nodes are also applied to parts in child nodes
  - Simple example: a robot arm
    - Base, lower arm, and upper arm
    - Base rotates ➔ lower and upper arm also rotate
    - Lower arm rotates ➔ upper arm also rotates

Simple Robot Arm Hierarchical Model





Use of Matrix Stacks in OpenGL to Implement Hierarchies

- Matrix stacks store projection & model-view matrices
- Push and pop matrices with:
  - glPushMatrix();
  - glPopMatrix();
- Can use to position entire object while also preserving it for drawing other objects
- Use in conjunction with geometrical transformations
- Example: Robot program

OpenGL Hierarchical Models

- Set up a hierarchical representation of scene (a tree)
- Each object is specified in its own modeling coordinate system
- Traverse tree and apply transformations to bring objects into world coordinate system
- Traversal rule:
  - Every time we go to the left at a node with another unvisited right child, do a push
  - Every time we return to that node, do a pop
  - Do a pop at the end so number of pushes & pops are the same

GLUT Animation

- Simple method is to use an “idle” callback
  - Called whenever window’s event queue is empty
  - Could be used to update display with the next frame of the animation
  - Identify the idle function with:
    - glutIdleFunc(idle_ftn())
  - Simple Example:

```
void idle_ftn()
        { glutPostRedisplay(); }
```

    - Posts message to event queue that client area needs to be repainted
    - Causes display callback function to be invoked
    - Effectively displays next frame of animation

Double Buffering

- Use two display buffers
- Front buffer is displayed by display hardware
- Application draws into back buffer
- Swap buffers after new frame is drawn into back buffer
- Implies only one access to display hardware per frame
- Eliminates flicker
- In OpenGL, implement by replacing glFlush() with glutSwapBuffers() in display callback
- In initialization function, must use:
  glutInitDisplayMode(GLUT_DOUBLE | GLUT_RGB);
- See anim_square & cone_anim examples