3-D Graphics

Overview of 3-D Computer Graphics

- Display image of real or imagined 3-D scene on a 2-D screen

Some Aspects of 3-D Graphics

- Modeling and Rendering
  - Wireframe Models
  - Polygon Mesh Models
- Rendering
  - Types of Projections
  - The Viewing Pipeline
  - Hidden surface removal
  - Shading

Problem # 1: Modeling

- Representing objects in 3-D space
- First need to represent points
- Use a 3-D coordinate system, e.g.:
  - Cartesian: (x, y, z)
  - Spherical: (rho, theta, phi)
  - Cylindrical: (r, theta, z)

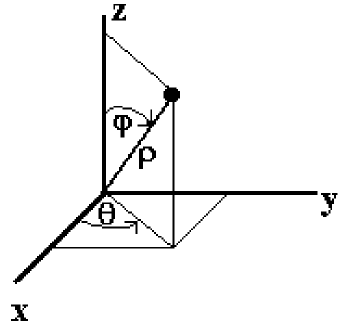
Conversions

- Spherical to Cartesian
  - $x = \rho * \sin(\phi) * \cos(\theta)$
  - $y = \rho * \sin(\phi) * \sin(\theta)$
  - $z = \rho * \cos(\phi)$

RH Coord System

Could be LH

Viewing system



Types of 3-D Models

- 1. Boundary Representation (B-Rep)
  - Surface descriptions
  - Two common ones:
    - A. Polygonal
    - B. Bicubic parametric surface patches
- 2. Solid Representation
  - Solid modeling

Polygonal Models

- Object surfaces approximated by a mesh of planar polygons

Scene -->

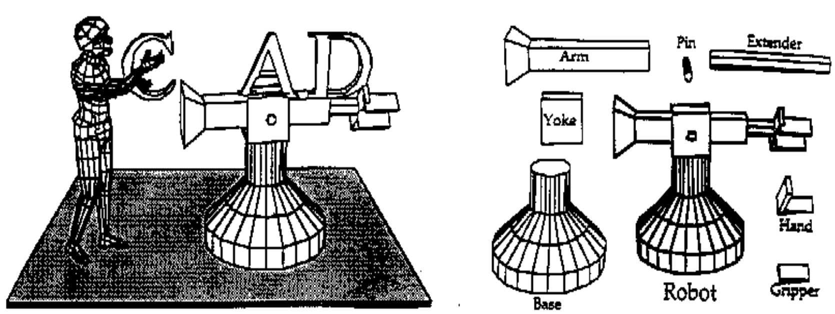
Objects -->

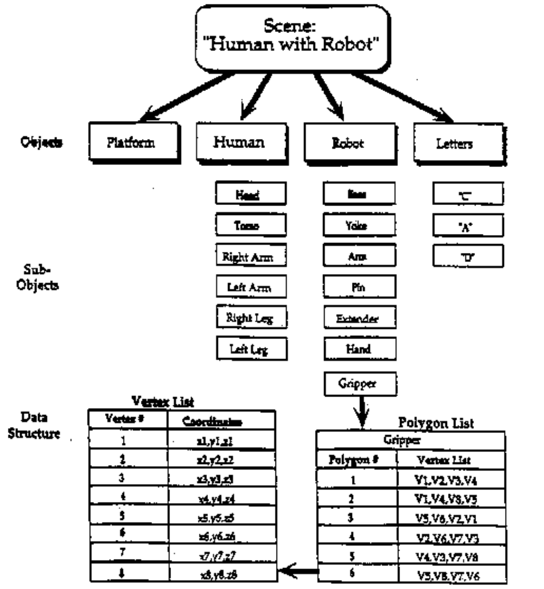
Sub-objects -->

Polygons -->

Vertices (points)

Polygon Mesh Model Example Scene

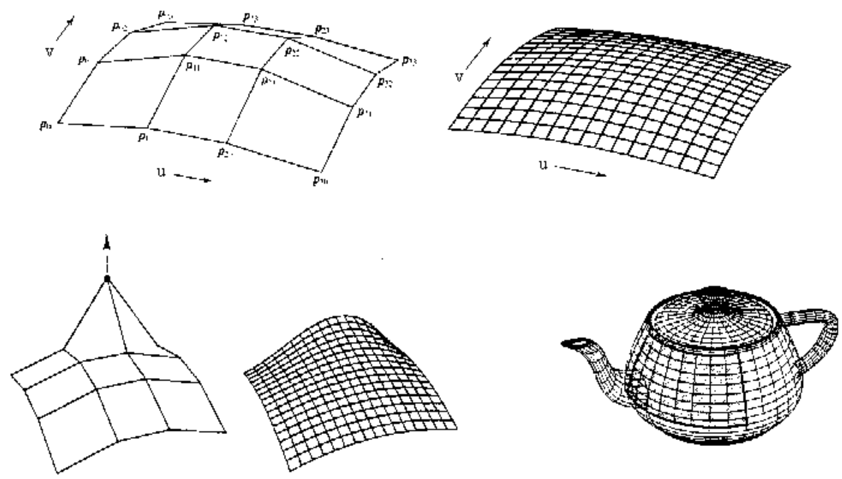




Bicubic Parametric Surface Patches

- Objects represented by nets of elements called surface patches
  - Polynomials in two parametric variables
  - Usually cubic
    - Bezier surface patches
    - B-Spline surface patches

Bicubic Parametric Surface Patches

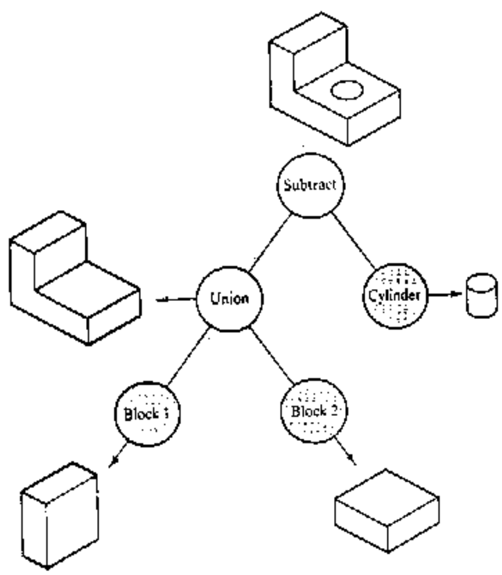


Solid Representation-- Solid Modeling

- Objects represented exactly by combinations of elementary solid objects
  - e.g., spheres, cylinders, boxes, etc
  - Called geometric primitives

Constructive Solid Geometry (CSG)

- Complex objects built up by combining geometric primitives using Boolean set operations
  - union, intersection, difference
- and linear transformations
- Object stored as a tree
  - Leaves contain primitives
  - Nodes store set operators or transformations

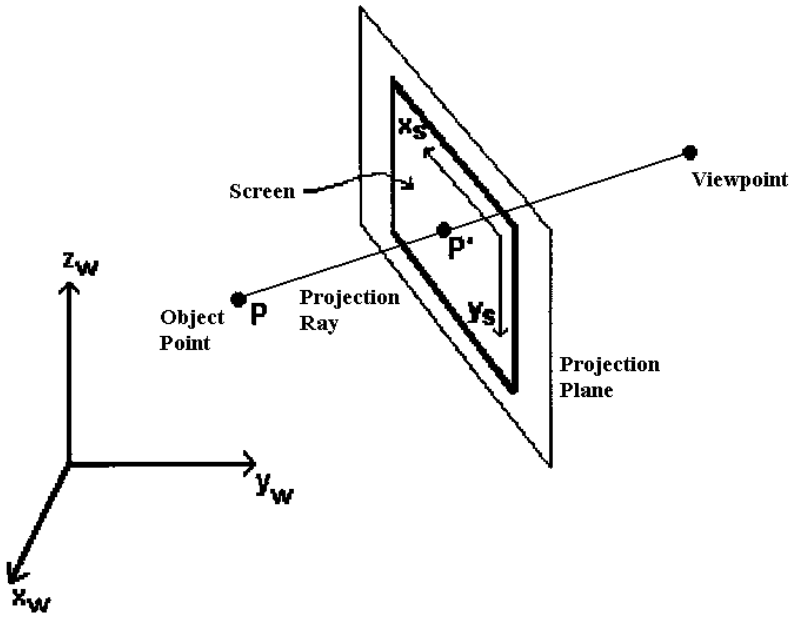


Problem # 2: Rendering

- Displaying a 2-D view of a 3-D model

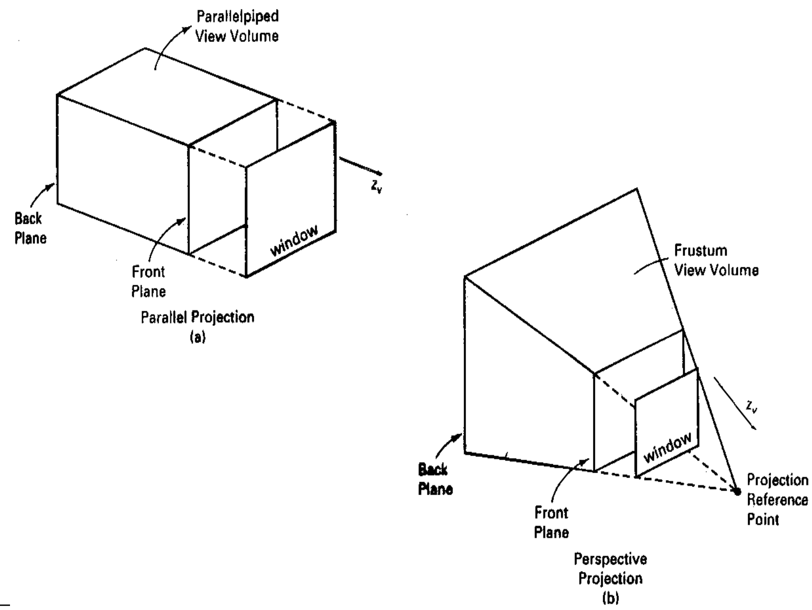
A. Projection

- Going from 3-D to 2-D
  - Every world coordinate point in scene (xw,yw,zw) maps to a point on device viewing screen (xs,ys)
- Camera model
  - Construct projection rays
    - From points in scene through projection plane terminating on “Center of Projection”
      - Camera point or view point
  - Projection Point:
    - Intersection of projection ray with projection plane



Two Basic Types of Projection

- 1. Parallel projection
  - Center of Projection at infinity
  - So projection rays are parallel
  - Equal-size objects at different distances from screen project to same size images
  - Parallel lines in scene project to parallel lines on screen
  - Useful in CAD



- 2. Perspective projection
  - Center of Projection at finite distance from screen
  - Far objects of the same size project to smaller images than close objects
    - Farther objects appear to be smaller
    - More realistic images
    - Parallel lines in scene don’t necessarily project to parallel lines on screen

B. Hidden surface removal

- Surfaces facing away from viewer are invisible
  - Should not be displayed
    - Backface culling
- Surfaces blocked by objects closer to viewer are invisible
  - Should not be displayed
    - General hidden surface removal algorithms

C. Shading

- Projections of surfaces should be colored (shaded)
- Color depends on intensity of light reflected from surface into viewer's eye
- Need an illumination/reflection model
  - Must take into account:
    - Material properties of surfaces
    - How light interacts with them

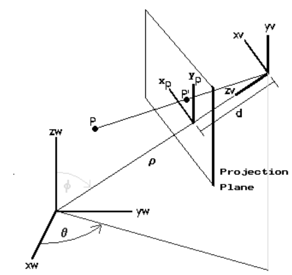
D. Other effects

- Shadows
- Transparency
- Multiple reflections
- Atmospheric absorption
- Surface textures
- Lots of others
- Physics and Optics!!

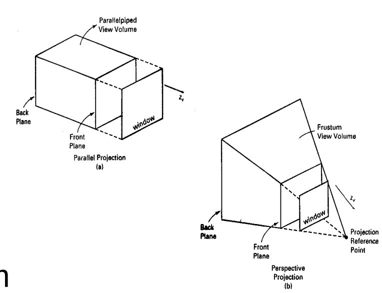
The Viewing Pipeline

- Chain of transformations/operations needed to go from a 3-D model to a 2-D image on the viewing screen

1. Local coordinate space (3-D):
   Individual object descriptions given
   |
   | Modeling Transformations
   | (Geometric transformations)
   v
2. World coordinate space (3-D):
   Scene is composed
   Objects, lights positioned
   |
   |
   | 3-D Viewing Transformation
   v



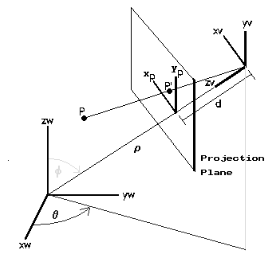
3. Viewing coordinate space (3-D):
   Eye/camera coordinate system
   |
   | 3-D clipping
   | Backface culling
   |
   v
4. 3-D viewing volume:
   Eye/camera coordinate system
   |
   | Projection Transformation
   |
   v



5. 2-D projection plane description:

2-D World coordinate system window

|
| 2-D Viewing xformation (window to viewport)
| 2-D clipping
| Hidden surface removal
| Shading
| Other effects
|
v



6. 2-D Device coordinate space:

2-D Screen coordinate system viewport

3-D Modeling with Polygons

- Two types of polygon models
  1. Wireframe
     - Store the polygon edges
     - List of edge endpoints
     - Not useful for shaded images
  2. Polygon Mesh
     - Store the polygon faces:
     - Array of vertex lists
     - One list for each polygon

Data structures

- Polygons represent/approximate object surfaces
- In either case we must store 3-D world coordinates of each vertex
  - Use an array of 3-D points:

```
struct  point3d {float x; float y; float z};
                                        // a single 3-D point
Struct  point3d  w_pts[ ];              // w_pts is the 3-D
                                        // points array
```

Storing Polygons in a Wireframe Model

- Store polygon edges as an array
- Each element a pair of indices into the 3D points array:

```
int edges[ ][2];  // Each second-index value gives the
                  // position of an edge's endpoint vertex
                  // in the 3-D points array
```

Storing Polygons in a <u>Polygon Mesh</u> Model

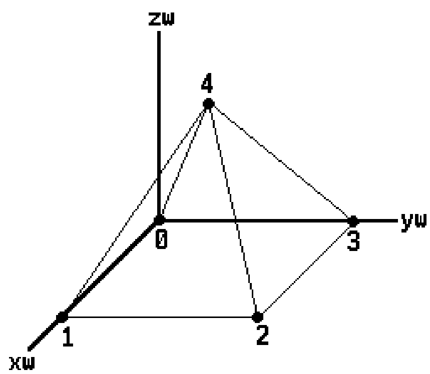
- Object: Can be represented as an array of polygons
- Each polygon consists of:
  - (a) the number of vertices in the polygon
  - (b) a list of indices into the 3-D points array
    - (An index gives the position of a vertex in the 3-D points array)

```
struct polygon {int n; int *inds};
        // n: The number of vertices
        // inds: List of indices into the points array
              // Specifies which vertices form the polygon

struct polygon object[ ];
                     // The object being modeled
                     // An array of polygons
```

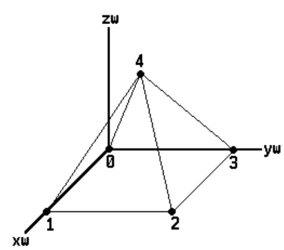
Example--A Pyramid

- Pyramid below has 5 vertices, 8 edges and 5 polygon faces



Vertex Coordinates

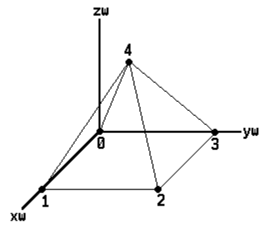
vertex	xw	yw	zw
0	0	0	0
1	150	0	0
2	150	150	0
3	0	150	0
4	75	75	150



The Pyramid's Points Array

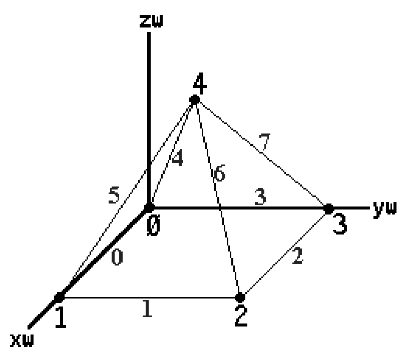
```
struct  point3d  w_pts[5];
                          // Pyramid vertices in world coords.
int b=150, h=75 ;    // Dimensions of pyramid

//  Set up world coordinate points array
w_pts[0].x=w_pts[0].y=w_pts[0].z=0;
w_pts[1].x=b; w_pts[1].y=w_pts[1].z=0;
w_pts[2].x=w_pts[2].y=b; w_pts[2].z=0;
w_pts[3].x=w_pts[3].z=0; w_pts[3].y=b;
w_pts[4].x=w_pts[4].y=b/2; w_pts[4].z=h;
```



Edge Array (Wireframe)

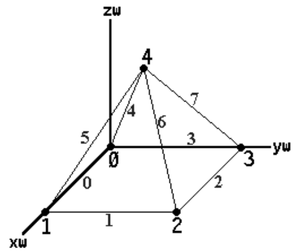
```
Edge   Endpoints
      (points array indices)
-----------------------------
 0      0, 1
 1      1, 2
 2      2, 3
 3      3, 0
 4      0, 4
 5      1, 4
 6      2, 4
 7      3, 4
```



Edge Array

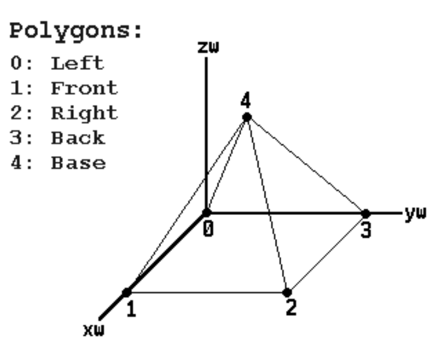
- Edge array could be generated by:

```
int edges[8][2] =
   {{0,1},{1,2},{2,3},{3,0},{0,4},{1,4},{2,4},{3,4}};
```



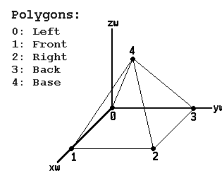
Polygons Array (Mesh)

polygon	# vertices	vertices
0	3	0,1,4
1	3	1,2,4
2	3	2,3,4
3	3	0,4,3
4	4	0,3,2,1



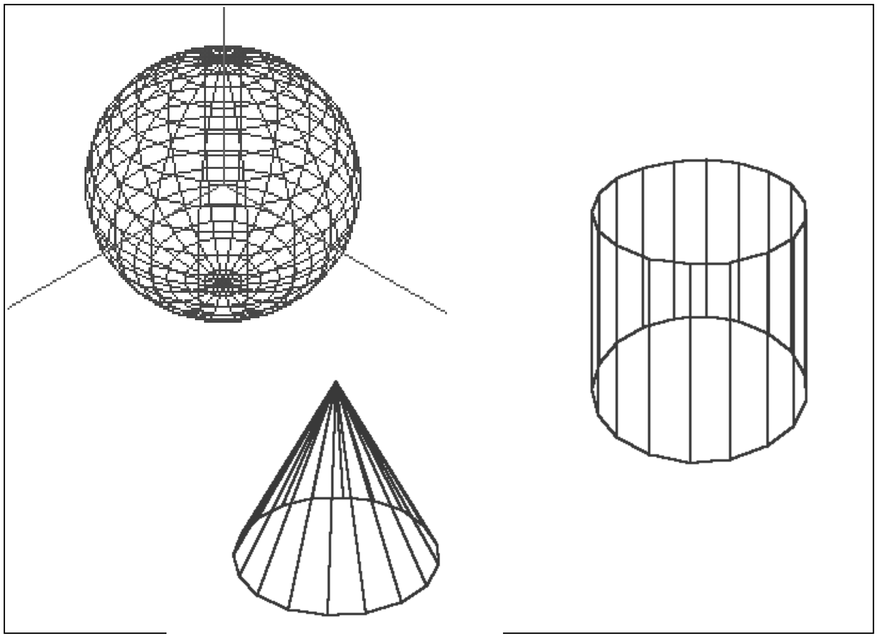
- Polygon array could be generated by:

```
struct polygon object[5];
// Allocate Space:
for (i=0;i<=3;i++)
    { object[i].n=3; object[i].inds = (int *) calloc(3,sizeof(int)); }
object[4].n=4; object[4].inds = (int *) calloc(4,sizeof(int));
// Define the polygons in the object
// define the side triangles
object[0].inds[0]=0; object[0].inds[1]=1; object[0].inds[2]=4;
object[1].inds[0]=1; object[1].inds[1]=2; object[1].inds[2]=4;
object[2].inds[0]=2; object[2].inds[1]=3; object[2].inds[2]=4;
object[3].inds[0]=0; object[3].inds[1]=4; object[3].inds[2]=3;
// define the square base
object[4].inds[0]=0; object[4].inds[1]=3;object[4].inds[2]=2;
object[4].inds[3]=1;
```



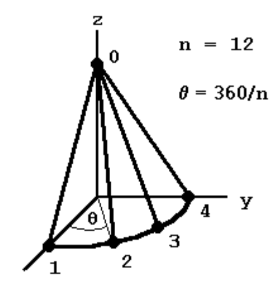
More Complex 3-D Objects

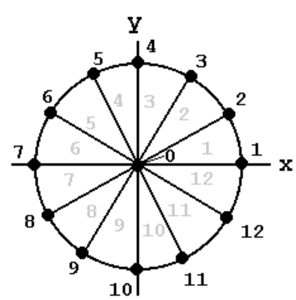
- Approximate surfaces with polygons
- Often points, edges, and/or polygons arrays can be generated procedurally



Example 1: A Cone

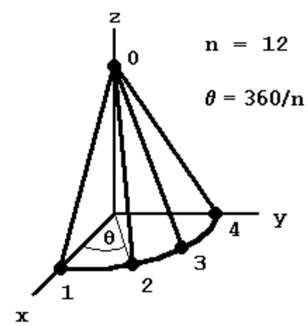
- Approximate with n triangular sides
- n+1 vertices (apex + n in the base)
- And a Base polygon with n sides (example, n=12)





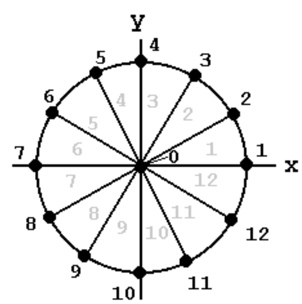
Cone Points Array

- Base points:
  x = R * cos (i * θ);
  y = R * sin (i * θ);
  // θ = 360/n
  z = 0;
- Apex point:
  x = y = 0;
  z = h; // (height of cone)



Cone Polygons Array

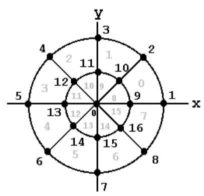
poly[0] = {12, {12,11,10,9,8,7,6,5,4,3,2,1}};
poly[1] = {3, {1,2,0}};
poly[2] = {3, {2,3,0}};
poly[3] = {3, {3,4,0}};
poly[4] = {3, {4,5,0}};
…
poly[12] = {3,{12,1,0}};

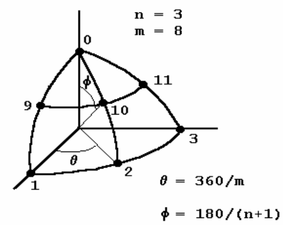


- The triangles can be generated in a loop

Example 2: A Sphere

- Divide with n lines of latitude and m lines of longitude
- Gives triangles and quadrilaterals
- Latitude/Longitude intersection points used as approximating-polygon vertices
- Number of vertices = m*n+2
- Number of polygons = (n+1)*m
- Example n=3, m=8





Example: n=3, m=8

8 * 3 + 2 = 26 vertices

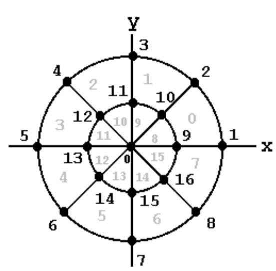
Can get x, y, z from spherical coordinates

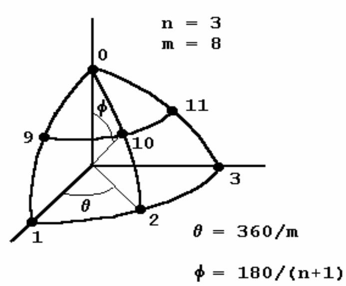
Loop j: 0->n-1 (latitudes),   i: 0->m-1 (longitudes)

$x = R * \sin(i*\theta) * \cos(j*\phi);$

$y = R * \sin(i*\theta) * \sin(j*\phi);$

$z = R * \cos(j*\phi);$





(3+1)*8 = 32 polygons

Number them in a consistent way

poly[0] = {4, {1,2,10,9}}; // Upper Hemisphere

poly[1] = {4. {2,3,11,10}};

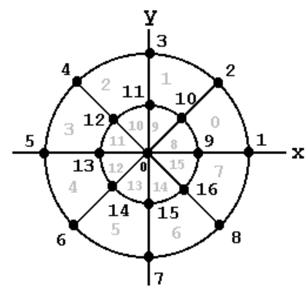
etc.

poly[8] = {3, {0,9,10}};

poly[9] = {3, {0,10,11}};

etc.

These can be generated in a loop



3-D Geometric Transformations

- Move objects in a 3-D scene
- Extension of 2-D Affine Transformations
- Three important ones:
  - Translation
  - Scaling
  - Rotations

Representing 3-D Points

- Homogeneous coordinates
- P (x,y,z) --> P' (x',y',z')

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \;\text{-->}\; \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

Translations

- Given 3-D translation vector T=(tx, ty, tz)
- Component equations

  x' = x + tx

  y' = y + ty

  z' = z + tz
- Represent translation as matrix equation

  P' = T * P
- T is a 4 X 4 Homogeneous Matrix

Homogeneous Translation Matrix

$$T = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Notice obvious extension from 2-D to 3-D

Scaling with respect to origin

- Given three scaling factors sx, sy, sz

  P' = S * P
- S is the following 4 X 4 scaling matrix:

$$S = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Again obvious extension from 2D

Rotations

- Need to specify angle of rotation
- And axis about which the rotation is to be performed
- Infinite number of possible rotation axes
  - Rotation about any axis: linear combinations of rotations about x-axis, y-axis, z-axis

Rotations about z-axis

- Consider rotation of point P=(x,y,z) by angle theta about the z-axis giving rotated point P'=(x',y',z')
  - Same x,y equations as in the 2-D case
  - z will not change

Z-Axis Rotation Component Equations

x' = x*cos(theta) - y*sin(theta)

y' = x*sin(theta) + y*cos(theta)

z' = z

- Represented as homogeneous matrix equation:

  P' = Rz * P

Z-Axis Rotation Matrix

$$
Rz = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Rx Matrix for rotations about x-axis

- Symmetry argument

```
        y          z
        |          |
        |          |
        |          |
         ------x    ------y
      /          /
     /          /
    z          x
 about z     about x
```

Make replacements:

x --> y
y --> z
z --> x

- Original rotation about z-axis equations:

  x' = x*cos(theta) - y*sin(theta)
  y' = x*sin(theta) + y*cos(theta)
  z' = z

- x->y, y->z, z->x transformed equations:

  y' = y*cos(theta) - z*sin(theta)
  z' = y*sin(theta) + z*cos(theta)
  x' = x

- Represented as matrix equation:

  P' = Rx * P

$$
Rx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(theta) & -\sin(theta) & 0 \\ 0 & \sin(theta) & \cos(theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Ry Rotation Matrix

- Symmetry:

```
           y            x
           |            |            Replacements:
           |            |                x --> z
           |            |                y --> x
            ------x      ------z         z --> y
         /            /
       /            /
     z            y
  about z       about y
```

x --> z

y --> x

z --> y

z' = z*cos(theta) - x*sin(theta)

x' = z*sin(theta) + x*cos(theta)

y' = y

P' = Ry * P

$$Ry = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Sense

- Positive sense
  - Defined as counter clockwise as we look down the rotation axis toward the origin