Viewing Transformation

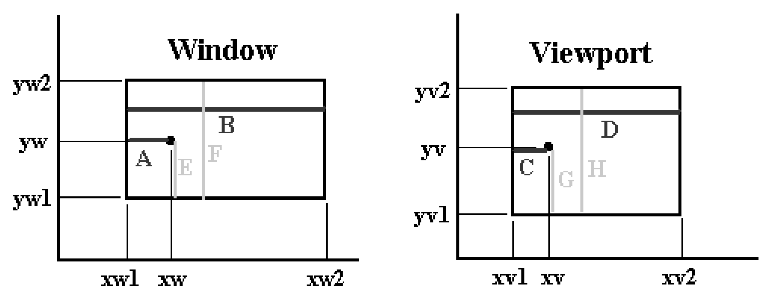
Clipping

2-D Viewing Transformation

2-D Viewing Transformation

- Convert from Window Coordinates to Viewport Coordinates
- (xw, yw) --> (xv, yv)
- Maps a world coordinate <u>window</u> to a screen coordinate <u>viewport</u>
- Window defined by: (xw1,yw1), (xw2,yw2)
- Viewport defined by: (xv1,yv1), (xv2,yv2)
- Basic idea is to maintain proportionality

Window to Viewport Transformation



A/B = C/D

$$\frac{xw-xw1}{xw2-xw1} = \frac{xv-xv1}{xv2-xv1}$$

xv=(Wv/Ww)*xw + xv1-(Wv/Ww)*xw1

where:
Ww=xw2-xw1 (window width)
Wv=xv2-xv1 (viewport width)

E/F = G/H

$$\frac{yw-yw1}{yw2-yw1} = \frac{yv-yv1}{yv2-yv1}$$

yv=(Hv/Hw)*yw+ yv1-(Hv/Hw)*yw1

where:
Hw=yw2-yw1 (window height)
Hv=yv2-yv1 (viewport height)

Viewing Transformation in Windows: Mapping Modes

Windows Viewing Transformation: Mapping Modes

- Create logical coordinate system
  - Define direction of axes
  - Define units
  - Can also move the origin
- Windows maps output to real device
  - e.g., plot at 100,100 "logical millimeters”
  - Windows figures out where on screen
  - Not exact, but close
- It’s Windows way of implementing the viewing transformation

Windows Mapping Modes

MAPPING MODE	LOGICAL UNIT	X-AXIS	Y_AXIS
MM_TEXT	Pixel	Right	Down
MM_HIENGLISH	.001 inch	Right	Up
MM_LOENGLISH	.01 inch	Right	Up
MM_HIMETRIC	.01 mm	Right	Up
MM_LOMETRIC	.1 mm	Right	Up
MM_TWIPS	1/20 point=1/1440”	Right	Up
MM_ISOTROPIC	Arbitrary (x==y)	Selectable	
MM_ANISOTROPIC	Arbitrary (x!=y)	Selectable	

Changing the Mapping Mode

- pDC->SetMapMode(MAP_MODE);
- Maps logical coordinates to device coordinates
  - Device Coordinate (physical)
    - units: pixels
    - +x: right, +y: down
  - Converts logical ("window") to device ("viewport") coordinates as follows

    xV = (xVExt/xWExt) * (xW - xWOrg) + xVOrg

    yV = (yVExt/yWext) * (yW - yWOrg) + yVOrg
- (xWOrg,yWOrg) and (xVOrg,yVOrg) are the origins of the window and viewport
- Both are (0,0) in the default device context

Moving Origins

- pDC->SetWindowOrg(x,y); // logical units
  - For x,y positive, think of this as moving the upper left-hand corner of the physical device viewport (screen) up and right by (x,y) logical units
- pDC->SetVieportOrg(x,y); // device units--pixels
  - For x,y positive, think of this as moving the lower left-hand corner of the logical window down and right by (x,y) device units
- Both move the coordinate system origin to (x,y), but units of x,y are different

Variable Unit Mapping Modes

- Coordinate axes can have any size/orientation
- MM_ISOTROPIC-- x & y units must be same size
- MM_ANISOTROPIC-- different x and y units
- Set the X and Y scaling factors with:

  pDC->SetWindowExt (xWExt, yWExt);

  pDC->SetViewportExt (xVExt, yVExt);
- X scaling factor in going from Logical Coordinates to Device Coordinates = xVExt/xWExt
- Y scaling factor = yVExt/yWExt

Example 1

- Create coordinate system where each logical unit is two pixels:
  - twice the default device unit coordinates

```
pDC->SetMapMode (MM_ISOTROPIC);
pDC->SetWindowExt (1, 1);
pDC->SetViewportExt (2, 2);
```

Example 2

- Create coordinate system with y-axis up, each y-unit = 1/4 pixel; x-axis unchanged:

```
pDC->SetMapMode (MM_ANISOTROPIC);
pDC->SetWindowExt (1, -4);
pDC->SetViewportExt (1, 1);
```

Example 3

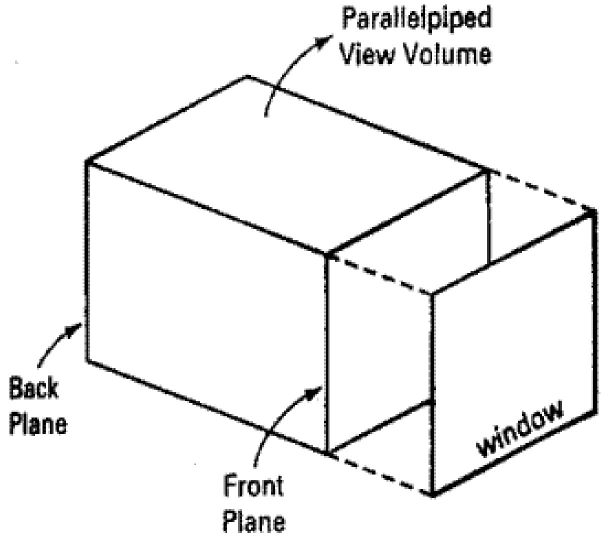
- Create coord system where client area is always 1000 units high & wide, y-axis up:

```
CSize size;
size = pDC->GetWindowExt (); // get client area size
   // returns size in default device units--here pixels
pDC->SetMapMode (MM_ANISOTROPIC);
pDC->SetWindowExt (1000, -1000);
pDC->SetViewportExt (size.cx, size.cy);
```

- Now (1000,1000) will always be at upper right edge of client area

OpenGL Viewing Transformation

- OpenGL designed for 3D graphics
- Must project onto 2D window
- Also do window to viewport transformation
  - with clipping
- For 2D graphics, use an orthographic projection
  - gluOrtho2D(xmin,xmax,ymin,ymax)
    - Equivalent to taking z=0 & setting a “window” with clipping boundaries: xmin<=x<=xmax, ymin<=y<=ymax -- logical units used
      - Will be mapped to entire client area of physical window
      - Client area determined by:
        - glutInitWindowSize(width,height)
        - Device units used



Parallel Projection

OpenGL Viewport

- gluOrtho2d(left,right,bottom,top) and glutInitWindowSize(w,h) map the “window” to the entire w X h client area
- glViewport(x,y,w,h) maps the “window” to the specified viewport within the client area
  - Device units used

Clipping

Clipping

- Elimination of parts of scene outside a window or viewport
- Clipping with respect to a window (Given: xwmin, ywmin, xwmax, ywmax)
  - Clip at this level ==> fewer points go through viewing transformation
- Clipping with respect to a viewport (Given: xvmin, yvmin, xvmax, yvmax)

Clipping

- Points
- Lines
  - Cohen-Sutherland Line Clipper
- Polygons
  - Sutherland-Hodgeman Polygon Clipper
  - Weiler-Atherton Polygon Clipper
- Other Curves
- Text

Point Clipping

- Given:
  - point (x,y)
  - clipping rectangle (window or viewport) (xmin,ymin,xmax,ymax)
- Point test:

```
if ((x<=xmax) && (x>=xmin)
    && (y<=ymax) && (y>=ymin)
        the point x,y lies inside the clip area
```

  - so keep it!

Line Clipping

- Could apply point test to all points on the line
  - Too much work
- Need a simple test involving the line's endpoint coordinates

Cohen-Sutherland Line Clipper

- Observation-- All lines fall into one of three categories
  1. Both endpoints inside clip rectangle
     - (Trivially accept entire line)
  2. Both endpoints outside clip rectangle on the same side of one of its borders
     - (Trivially reject entire line)
  3. Neither 1 nor 2
     - (Chop off part of line outside one of borders and repeat)

Region Code

- A tool in assigning lines to Category 1 or 2
- 4-bit region code number assigned to an endpoint (x,y)
- Any set bit means endpoint is outside of one of the 4 borders of the clip rectangle
- Each bit position corresponds to a different border

Region Code RC = LRBT

- L=left (if x<xmin, L=1, else L=0)
- R=Right (if x>xmax, R=1, else R=0)
- B=Bottom (if y<ymin, B=1, else B=0)
- T=Top (if y>ymax, T=1, else B=0)
- The Region Code Divides the entire x-y plane 9 regions

Region Codes (LRBT)

```
          |         |
  1001    |  0001   |   0101
          |         |
 ---------+---------+---------
          |         |
  1000    |  0000   |   0100
          |         |
 ---------+---------+---------
          |         |
  1010    |  0010   |   0110
          |         |
```

Category 1 Lines

- Assume region codes for the line's endpoints are RC1 and RC2
- Take Boolean OR of two region codes

  if (RC1 | RC2 == 0)

  both RCs are 0000

  both endpoints are inside

  so it's Category 1 (trivial accept)

Category 2 Lines

- Both endpoints are outside same border
  - (Category 2 line)
- Then both region codes will have the same bit set in one of the four bit positions
  - Boolean AND will give a non-zero result:

    if (RC1 & RC2 != 0)
    - both endpoints are outside same border
    - so it's Category 2 (trivial reject)

Category 3 Lines

- Want to chop off outside part of line
- May have both endpoints (P1 & P2) outside different borders of clip region
  - So it's not important which end is chopped off first
- But if one endpoint's in and other's out:
  - Want to chop off the outside end
  - So Arrange things so P1 is the outside point
    - (swap P1 & P2 if necessary)

How to do the Chopping

- Want to determine the new endpoint
- Endpoint coordinates (x1,y1), (x2,y2) are known
- Slope m can be computed from them
- So y = m*(x-x2) + y2 (point slope form)
- Or x = (y-y2)/m + x2
- Look at P1's region code (RC1)
- Four possible cases:

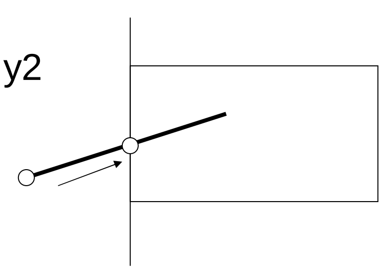
If RC1 == 1xxx (P1 to left of xmin)

- New endpoint should be on the left boundary:

  x1 <-- xmin

  y1 <-- m*(xmin-x2) + y2

  Reset RC's L bit



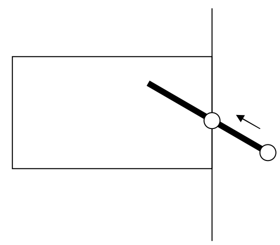
If RC1 == x1xx (P1 right of xmax)

- New endpoint should be on the right boundary:

  x1 <---xmax

  y1 <---m*(xmax-x2) +y2

  Reset RC's R bit



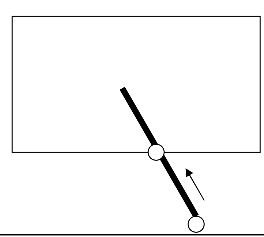
If RC1 == xx1x (P1 below ymin)

- New endpoint should be on the bottom boundary:

  y1 <---ymin

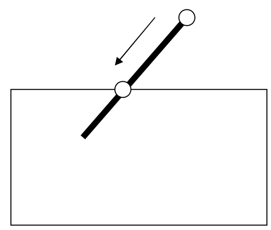
  x1 <---(ymin-y2)/m + x2

  Reset RC's B bit



If RC == xxx1 (P1 above ymax)

- New endpoint should be on the top boundary:
  - y1 <---ymax
  - x1 <---(ymax-y2)/m + y2
  - Reset RC's T bit



- Horizontal and vertical lines are special cases
  - Horizontal:
    - y doesn’t change and x = xboundary
  - Vertical:
    - x doexn’t change and y = yboundary

The C-S Line Clipping Algorithm

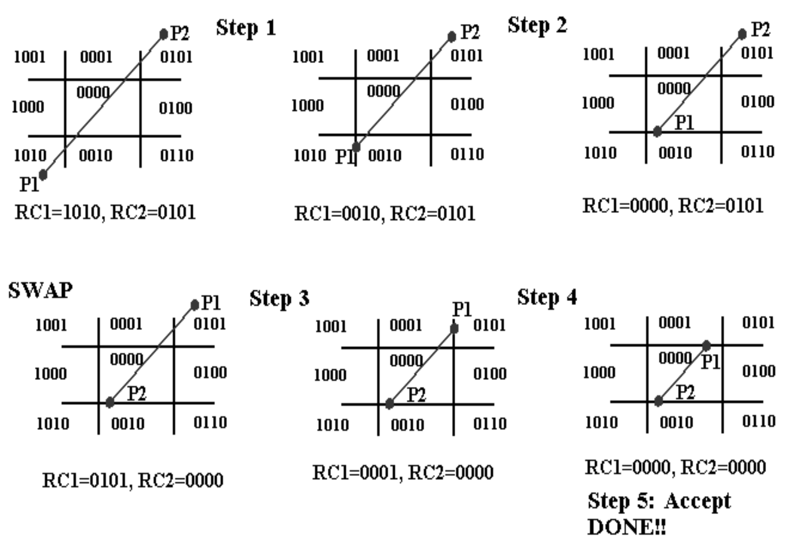
- Input:
  - Original endpoints (x1,y1,x2,y2)
  - Clip region boundaries (xmin,ymin,xmax,ymax)
- Output:
  - Accept Code (AC)
    - AC==TRUE ==> some part of line was inside
    - AC==FALSE ==> no part of line was inside
  - Clipped Line endpoints (x1,y1,x2,y2)
    - only if AC==TRUE

C-S Algorithm Pseudo-code:

```
CS_LineClip(xmin,ymin,xmax,ymax,x1,y1,x2,y2,AC)
   done = FALSE
   While (!done)
      Calculate endpoint codes rc1, rc2
      If ((rc1 | rc2) == 0 )    // Category 1
         done = TRUE
         AC = TRUE
      Else
         If ((rc1 & rc2) != 0)  // Category 2
            done = TRUE
            AC = FALSE
         Else
            If (P1 is inside)
              Swap (x1,y1), (x2,y2); and rc1,rc2
```

```
If (L-bit of rc1 is set)     // 1xxx
  x1 = xmin
  y1 = m*(xmin-x2) + y2
Else
  If (R-bit of rc1 is set)   // x1xx
    x1 = xmax
    y1 = m*(xmax-x2) + y2
  Else
    If (B-bit of rc1 is set) // xx1x
      y1 = ymin
      x1 = (ymin-y2)/m + x2
    Else                     // xxx1
        y1 = ymax
        x1 = (ymax-y2)/m + x2
```

Cohen-Sutherland Clipping Example



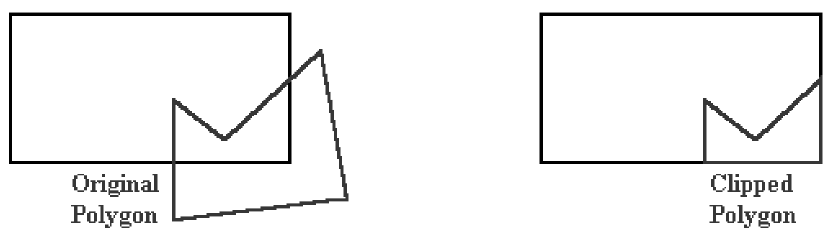
Polygon Clipping

Polygon Clipping

- Clip a polygon to a rectangular clip area
- Input
  - Ordered list of polygon vertices (nin, vin[ ])
  - Clip rectangle boundary coordinates (xmin, ymin, xmax, ymax).
- Output:
  - An ordered list of clipped polygon vertices (nout, vout[ ]).
  - vin[ ] and vout[ ] could be arrays of POINTs

Approaches to Polygon Clipping

- Use a line clipper on each polygon edge???
- But we usually won't get back a polygon
  - Parts of the clip rectangle will be edges of the clipped polygon that line clipper won't get
- Really need new list of edges (or vertices)



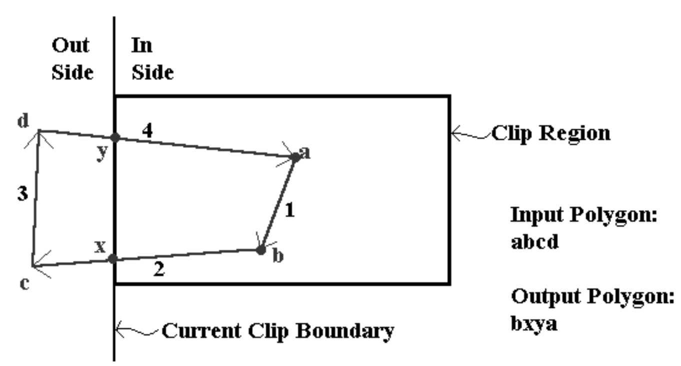
Sutherland-Hodgeman Polygon Clipper

- Approach:
  - Clip all polygon edges with respect to each clipping boundary
  - Do four passes; on each pass:
    - Traverse current polygon and clip with respect to one of the four boundaries
    - Assemble output polygon edges as you go
    - vin[ ] --> Clip Left --> vtemp1[ ] --> Clip Right --> vtemp2[ ] --> Clip Bottom --> vtemp3[ ] --> Clip Top --> vout[ ]

- On any polygon traversal the clip boundary divides plane into "in" side and "out" side
- For any given edge (vertices i and i+1),
  – during traversal, there are four possibilities:
  – (Assume vertex i has already been processed)

VERTEX i	VERTEX i+1	ACTION
in	in	Add Vertex i+1 to output list
out	out	Add no vertex to output list
in	out	Add intersection point with edge to output list
out	in	Add intersection point with edge and vertex i+1 to output list

Sample Traversal



Traversal		Type	Action
1	a → b	in-in	Add point b
2	b → c	in-out	Add intersection point x
3	c → d	out-out	Add nothing
4	d → a	out-in	Add intersection point y and point a

Implementation

- Function sh_clip()
  - Will clip an input polygon (ni, vi[ ])
  - With respect to a given boundary (bndry)
  - Generating an output polygon (no, vo[ ])
- Enumerate the boundaries as:
  - LEFT, RIGHT, BOTTOM, and TOP

sh_clip(ni, vi[ ], no, vo[ ], xmin, ymin, xmax, ymax, bndry);

vi[ ] and vo[ ]: could be arrays of POINTs

ni, no: number of points in each array

xmin, ymin, xmax, ymax: clip region boundaries

Using sh_clip() to clip a polygon

- Make four calls to sh_clip():

```
sh_clip(nin, vin[ ], ntemp1,  vtemp1[ ], xmin, ymin,
          xmax, ymax, LEFT);
sh_clip(ntemp1, vtemp1[ ], ntemp2, vtemp2[ ], xmin,
          ymin, xmax, ymax, RIGHT);
sh_clip(ntemp2, vtemp2[ ], ntemp3, vtemp3[ ], xmin,
          ymin, xmax, ymax, BOTTOM);
sh_clip(ntemp3, vtemp3[ ], nout, vout[ ], xmin, ymin,
          xmax, ymax, TOP);
```

Three Helper Functions

BOOL inside(V, xmin, ymin, xmax, ymax, Bndry)

- Returns TRUE if vertex point V is on the "in" side of boundary Bndry

intersect(V1, V2, xmin, ymin, xmax, ymax, Bndry, Vnew)

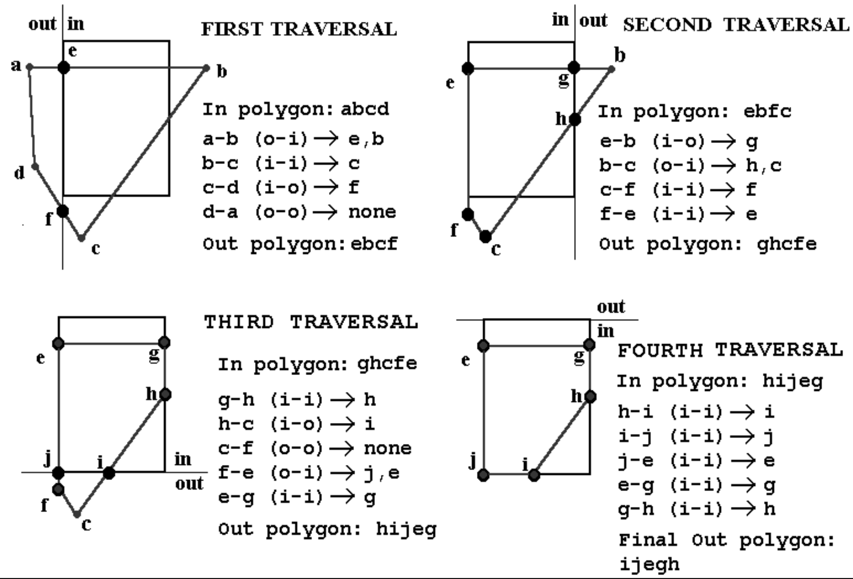
- Computes intersection point of edge whose endpoints are V1 and V2 with boundary Bndry
- Returns the resulting point in Vnew

output(V, n, vout[ ])

- Adds vertex point V to the polygon (n, v[ ])
  - n will be incremented by 1
  - vertex V added to end of polygon's vertex list v[ ]

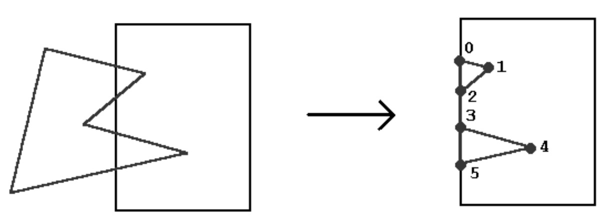
```
sh_clip (ni, vi[], no, vo[], bndry)
no = 0                          // output list begins empty
First_V = vi[0]                 // first vertex (i)
For (j=0 to ni-1)               // traverse polygon
  Second_V = v[(j+1) % ni]      // second vertex (i+1)
  If (inside(First_V, bndry)
    If (inside(Second_V, bndry)  // "in-in" case
       output(Second_V, no, vo)
    Else                         // "in-out" case
       intersect(First_V, Second_V, bndry, Vtemp)
       output (Vtemp, no, vo)
  Else
    If (inside(Second_V, bndry)  // "out-in" case
       intersect(First_V, Second_V, bndry, Vtemp)
       output(Vtemp, no, vo)
       output(Second_V, no, vo)   // no "out-out" case
  First_V = Second_V             // prepare for next edge
```

Example of S-H Clipping



Sutherland-Hodgeman Problems

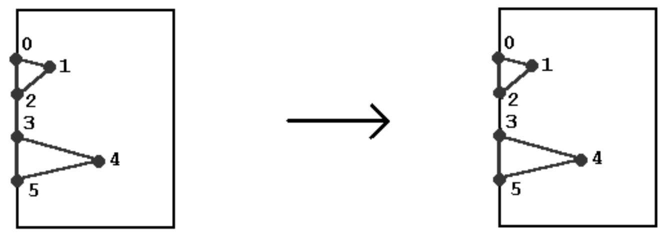
- Works fine with convex polygons
- But some concave polygons problematic
  - Extraneous edges along a clip boundary may be generated as part of the output polygon
  - Could cause problems with polygon filling



Output Polygon: 0,1,2,3,4,5
Extraneous Edge: 2-3

Solutions to S-H Problems

- Add a postprocessing step
  - Check output vertex list for multiple (>2) vertex points along any clip boundary
  - Correctly join pairs of vertices



Polygon: 0,1,2,3,4,5
0,2,3,5 are vertices
along the left boundary

So break into two polygons:
0,1,2 and 3,4,5

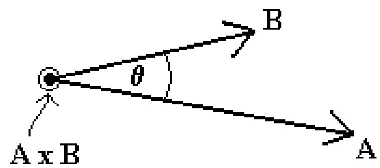
Other Solutions

- Add a preprocessing step
  - Split concave polygon into convex polygons
- Or use a more general clipping algorithm
  - For example, the Weiler-Atherton polygon clipper

Splitting Concave Polygons

- Split into convex polygons
- Use edge vector cross products

Vector Product of Two Vectors



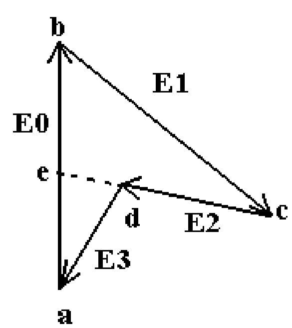
- V = A X B
- |V| = |A| |B| sin(θ)
- Direction: RH Rule
- In terms of components

```
             | i  j  k  |
A X B  =     | Ax Ay Az |
             | Bx By Bz |
i, j, k: unit vectors in x, y, z directions
```

Splitting Concave Polygons

- Process edges in clockwise order
- Form successive edge vectors
- Compute vector cross product between successive edge vectors
- If all cross products are not negative
  - Polygon is concave
  - Split it along line of first edge vector in the cross-product pair:
    - Compute intersections of this line with other edges
    - This splits polygon into two pieces
- Repeat this until no other edge cross products are positive

Splitting Concave Polygons



abcd --> aed & ebc

E0 X E1 --> -k
E1 X E2 --> -k
E2 X E3 --> +k ==> find int. pt.
E3 X E0 --> -k

$$E0 \times E1 = \begin{vmatrix} i & j & k \\ Dx0 & Dy0 & 0 \\ Dx1 & Dy1 & 0 \end{vmatrix}$$

$Dx0 = xb - xa$

$Dy0 = yb - ya$

Splitting Convex Polygon into Triangles

- Often convenient since triangles are the simplest polygon
  1. Define a sequence of three consecutive vertices to be a new polygon (triangle)
  2. Delete middle vertex from original vertex list
  3. Continue to form triangles until original polygon has only three vertices

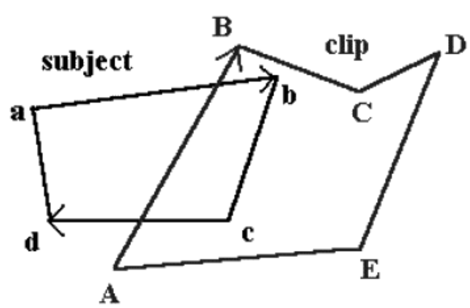
Weiler-Atherton Polygon Clipper

- Clips a "Subject Polygon" to a "Clip Polygon”
- Both polygons can be of any shape
- Result: one or more output polygons that lie entirely inside the clip polygon
- Basic idea:
  – Follow a path that may be a subject polygon edge or a clip polygon boundary until you get back to the starting vertex

The Weiler-Atherton Algorithm

1. Set up vertex lists for <u>subject</u> and <u>clip</u> polygons
   Ordering: as you move down each list, inside of polygon is always on the right side (clockwise)
2. Compute all intersection points between subject polygon and clip polygon edges
   - Insert them into each polygon's list
   - Mark as intersection points
   - Mark "out-in" intersection points
     (subject polygon edge moving from outside to inside of clip polygon edge)

Intersection Points & out-in Marking (General)



$\overline{ab}$:
x = xa + (xb-xa)t
y = ya + (yb-ya)t

$\overline{AB}$:
x = xA + (xB-xA)s
y = yA + (yB-yA)s

Solve for s and t
0<=t<=1 and 0<=s<=1 ==> Intersection Point

Vector cross product:
$\overline{ab}$ X $\overline{AB}$ = +k ==> Out-In
$\overline{cd}$ X $\overline{AB}$ = -k ==> In-Out

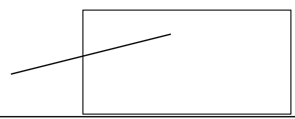
Intersection Points and Out-In Marking (Simple)

- If clip polygon is a rectangle:
  - Use point in/out test
  - e.g., for intersection with left boundary:

    x<xmin means outside, x>=xmin means inside
- Intersections also easy
  - Use Cohen-Sutherland ideas
    - e.g., for intersection with left boundary:

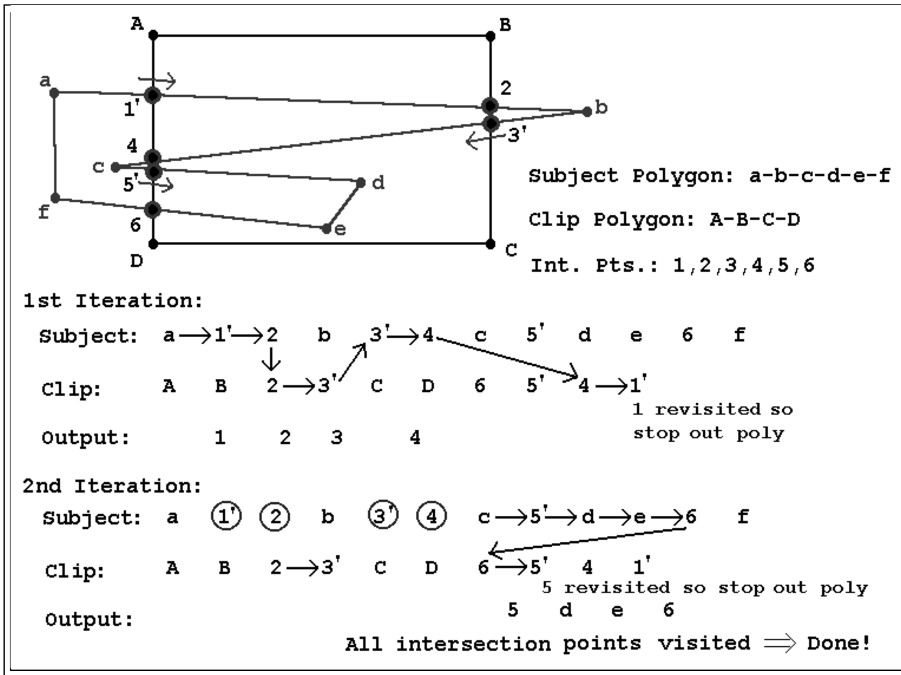
      x = xmin

      y = m*(xmin-x1) + y1



Weiler-Atherton Algorithm, continued

3. Do until all intersection points have been visited:
   - Traverse subject polygon list until a non-visited out-in intersection point is found;
   - Output it to new output polygon list
   - Make subject polygon list be the active list
   - Do until a vertex is revisited:
     - Get next vertex from active list & output
     - If vertex is an intersection point,
       - make the other list active
   - End current output polygon list



Clipping Other Curves

- Must compute intersection points between curve and clip boundaries
- In general solve nonlinear equations
- Many times approximation methods must be used
- Time consuming

Clipping Text

- Use successively more expensive tests

  1. Clip string

     Embed string in rectangle

     Clip rectangle (4 point tests)

     • entirely in ==> keep string

     • entirely out==>reject string

     • neither==>next test

2. Clip each Character

   Embed character in rectangle

   Clip rectangle (4 point tests)

   • entirely in ==> keep character

   • entirely out==>reject character

   • neither==>next test

3. Two possibilities for Character Clipping

   – Bitmapped: look at each pixel

   – Stroked: Apply line clipper to each stroke