

The first question is required; pick two questions from the “easy” group, two questions from the “medium” group, and one question from the “hard” group. Please put no more than one answer on a page, and indicate which question you are answering.

REQUIRED: Give formal definitions for O, Omega, Theta, P, and NP.

EASY QUESTIONS: PICK ANY TWO

EASY 1: Describe briefly what is meant by “optimal substructure.” You should describe a problem (and its solution) that exhibit optimal substructure.

EASY 2: Sketch pseudocode for an algorithm that has different functions for Big-O and Omega. The algorithm doesn’t need to do anything useful (i.e., it doesn’t have to be something that you learned in the Cormen book); it just needs to illustrate different computation complexity behavior for best and worst case.

EASY 3: What is the Big-O complexity for the following function?

```
int f2(int n)
{
    int i, x;

    if (n <= 1) return 1;

    x = n;
    for (i = 0 to n)
        x = x + i;
    return x + f2(n/2) + f2(n/2);
}
```

EASY 4: What impact can a programming language have on the computational complexity of an algorithm?

MEDIUM QUESTIONS: PICK ANY TWO

MEDIUM 1: Your company is building an airplane, and there are several subassemblies that need to be constructed; some of them must be completed before others can begin. For example, attachment of wings takes 3 days, but that cannot begin until both the wings and the fuselage are complete. Design an algorithm to determine the earliest date that the airplane can be completed, assuming you have enough employees to work on all parts at the same time.

MEDIUM 2: Google Maps can give you driving directions to go from Binghamton to New York City. What algorithm do they likely use? Sketch pseudocode for it.

MEDIUM 3: Assume you have 3 blue blocks, and 4 red blocks. How many different orderings of the blocks can you produce? For example, bbbRRRR is one ordering, RbbRbRR is another, and RRbbbRR is a third. There are seven blocks; consider the blocks of any given color, so the answer is *not* seven factorial.

MEDIUM 4: We're in Las Vegas, using a slot machine. It has three wheels, each with 8 different values (Elvis, Horseshoe, 7, Dollar Sign, Hearts, Clubs, Spades, Diamonds). The machine takes one quarter, and pays out one quarter if two of the values match, and one dollar if all three match. What are the odds that we'll get a quarter back? What are the odds that we'll get a dollar back?

HARD QUESTIONS: PICK ANY ONE

HARD 1: Describe the MAX-FLOW MIN-CUT theorem. There are three conditions that are true for a maximum flow in a network, and if any of the conditions are true, they all must be true. State what those conditions are.

HARD 2: Describe the boolean satisfiability problem. Why is it important? What might you be able to do if you had a solution for it?

HARD 3: Assume you have 3 red, 2 blue, and 2 yellow blocks. How many different orders are there? Show your work (and show the easy, fast way to do it!).

HARD 4: Suppose you have n unique integers; you could place them into a binary search tree in many different ways, making many different trees. As a binary search tree, the numbers must be arranged such that smaller values are to the left, and larger ones are to the right -- but how the tree branches can be changed. All numbers in the tree are at the leaves. Describe a recursive formulation to calculate the number of possible different trees, and give dynamic programming pseudocode to calculate that number.