

PhD Exam – Early Summer 2003 – Algorithms
Answer all five questions.

1. Give formal definitions for O , Ω , Θ , P , and NP .
2. The MAX-FLOW MIN-CUT theorem gives three conditions for a maximum flow network. If any of the three conditions is true, all must be true. State what these conditions are.
3. In matrix multiplication, we cannot change the order of the matrices, but we can choose which pair to multiply first. For example, $m_1 \times (m_2 \times m_3)$ and $(m_1 \times m_2) \times m_3$ each give the same result, but may require different amounts of work. If we want to multiply matrix m_i (which is p_i by q_i) by matrix m_{i+1} (which is p_{i+1} by q_{i+1} , and $q_i = p_{i+1}$), we need to do $p_i \times q_i \times q_{i+1}$ math operations. Sketch the dynamic programming algorithm to determine the minimum number of math operations needed to multiply n matrices.
4. Suppose we want to compress a file, and do a frequency count of the letters in the file. The letters, and their frequency, are given in the table below. Construct an optimal Huffman code for the letters, so that we get the maximum compression. Show the Huffman tree, and assuming we have a total of 46 characters in the file, how many bits are in the compressed file (ignore the bits required to describe the encoding tree).

A	12
B	11
C	9
D	8
E	6

5. Sketch pseudocode for Prim's minimum spanning tree algorithm.