Exam Questions

1. In the manufacture of computer circuit boards, it is common to have a machine that automatically solders wires to various points. This machine has a spool of wire, and under computer control, moves to one spot on the circuit board and connects one end of a wire; it then moves to another spot and attaches the other end of the wire. Normally, there are thousands of wires that need to be connected, so the machine has a lot of work to do.

If the machine always moves at a constant speed, is it possible to change the order that connections are made so that it takes the minimum time possible to complete the wiring of a circuit board? Either give an optimal (polynomial time) algorithm for this, or explain why one is not possible. If it is not possible, provide an algorithm within at least a constant factor of optimal.

2. Suppose you have a set of train cars on a track that splits into a “Y,” as shown in the figure. Design an algorithm to rearrange the cars from a random starting order, into an order where they are arranged by increasing number. Make your algorithm as efficient as you possibly can, minimizing the amount that you move the train cars; your algorithm is allowed to use any polynomial time algorithm to determine how the cars should move for “free.” What is the complexity of your algorithm for the car movement, and for the algorithms used to determine how you should move the cars?

3. Assume you have access to flight costs between all major cities, and have employees in your company scattered all over the country. Explain an algorithm to determine what city you should hold your company-wide meeting in, if you want to minimize the expenses of sending everyone to that city.

4. Describe a dynamic-programming approach to determine the minimum number of coins required to make change for a given amount (assuming the coins have arbitrary values; for example, 1, 7, 11, 36, 51, rather than the US denominations of 1, 5, 10, 25). This algorithm is “pseudopolynomial;” why?

5. Your company is building an airplane, and there are several subassemblies that need to be constructed; some of them must be completed before others can begin. For example, attachment of wings takes 3 days, but that cannot begin until both the wings and the fuselage are complete. Design an algorithm to determine the earliest date that the airplane can be finished, assuming you have enough employees to work on all parts at the same time.

6. Optimizing compilers frequently rearrange the orders of some instructions when generating machine code. Why do they do this? Does this change the big-O complexity of an algorithm?
Figure 1: Train cars in an arbitrary starting order (and there may be more than four cars). This figure is only to illustrate the problem, and your answer should not be specific to this instance.