Radiosity Method
(1) Concept

- Ray tracing:
  - synthesized (not very realistic as compared to results using the radiosity method)
  - it only models one aspect of the light interaction, which has perfect specular reflection and transmission
  - ambient constant to model

- Radiosity
  - interaction between diffusely reflecting surfaces, devised by the group in Cornell University (1980’s)
  - more reality than the ray-tracing method
  - successful in dealing with man-made environment (e.g., interiors of offices, factories and the like, closed environment)
  - Drawback: very computation intensive (very time-consuming) (e.g., simulating a steel mill which contains 30,000 patches costs 190 hours on VAX8700 machine).
(2) Theory

- Principle of heat transfer or interchange between surfaces

- The energy input to the system is from those surfaces that act as emitters
  - The light source is treated like any other surface except that it possesses an initial (non-zero) radiosity

- Divide the environment into “patches”

- Radiosity $B$ of a patch is a total rate of energy leaving a surface
  \[ \Rightarrow = \text{emitted energy} + \text{reflected energy} \]

- \textit{Reflected energy} = \textit{reflection coefficient} \times \textit{energy incident on the patch from all other patches}
e.g.,

- Energy exchange between patch $P_i$ and $P_j$ is a function of distance and orientation.
- High energy interchange will occur if $P_i$, $P_j$ are close together and parallel to each other.
- Radiosity is view-independent.
(3) Algorithm

\[ B_i dA_i = E_i dA_i + P_i \left( \int B_j dA_j \right) F_{dA_j dA_i} \]

\( E_i \): energy emitted from a patch per unit time and unit area \( dA_i \)

\( F_{dA_j dA_i} \): form factor between the \( dA_j \) and \( dA_i \) or the fraction of energy leaving \( dA_j \) that arrives at \( dA_i \)

In a close environment an energy equilibrium must be established itself and a set of linear equations is formulated.

For \( n \) patches:

\[ B_i A_i = E_i A_i + P_i \sum_{j=1}^{n} B_j F_{ji} A_j \quad (1) \]

\[ F_{ji} = F_{Aj Ai} \]

energy exchange depends only on the relative geometry of the patches:
\[ F_{ij} A_i = F_{ji} A_j \]
\[ \Rightarrow F_{ij} = F_{ji} \frac{A_i}{A_j} \]

substitute equation 1:
\[ \Rightarrow B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij} \text{ for each patch} \]

For n patched
\[ \Rightarrow n \text{ simultaneous equations in } n \text{ unknown } B_i: \]
\[
\begin{bmatrix}
1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\
-\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
-\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix} =
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

\( E_i \) values are nonzero only at surfaces that provide illumination.
The equation set is an expression of energy equilibrium for a particular wavelength; $E_i$ and $P_i$ are the function of wavelength.

⇒ A set of equation needs to be solved for each color band of interest (e.g., R, G, B or more)

- $F_{ii} = 0$ for a plane or convex surface – none of the radiation leaving a surface will strike itself.

- Radiosity values are constant over the extent of a patch.

- Standard render requires vertex radiosities or intensities.
(4) Calculation

- “Gathering” iteration
  - update the radiosity of single patch

- “Shooting” iteration
  - progressive refinement – update the entire image as each iteration

- Increasing accuracy of the solution:
  - “substructuring” (“elements” of a patch)
  - patches that need to be divided into elements are revealed by examine the graduation of the coarse patch solution

(5) Stages in a complete radiosity solution
(6) Form Factor definition

- calculation is non-trivial

- Solid angle subtended by $dA_j$ at $dA_i$
  
  \[
  dw_{ij} = \cos(\phi_j) \frac{dA_j}{r^2}
  \]
  
  $r$: distance between $dA_j$ and $dA_i$

- Energy leaving $dA_i$ that reaches $dA_j$
  
  \[
  dE_i dA_i = I_i \cos(\phi_i) dw_{ij} dA_i
  = I_i \cos(\phi_i) \cos(\phi_j) dA_i \frac{dA_j}{r^2}
  \]

- Total energy leaving $dA_i$ in all direction into the hemisphere centered on that area:
  
  \[E_i dA_i\]
• element → element

\[ F_{dA_i dA_j} = \frac{\text{Radiative energy reaching } dA_j \text{ from } dA_i}{\text{Total energy leaving } dA_i \text{ in all directions}} \]

Total energy leaving \( A_i \): \( E_i dA_i = I_i \pi dA_i \)
where
\( I_i \): intensity or energy
\( I_i \pi \): integrating over hemisphere centered on \( dA_i \)

\[ \Rightarrow F_{dA_i dA_j} = \frac{dE_i dA_i}{E_i dA_i} = \frac{\cos \phi_i \cos \phi_j dA_j}{\pi r^2} \]

\( dA_i \rightarrow A_j \):
\[ F_{dA_i dA_j} = \int_{A_j} \frac{\cos \phi_i \cos \phi_j dA_j}{\pi r^2} \]

• patch → patch

\[ F_{dA_i dA_j} = F_{ij} = \frac{\text{Radiative energy leaving surface } A_i \text{ that strikes } A_j \text{ directly}}{\text{Energy leaving } A_i \text{ in all directions of hemispherical space surrounding } A_i} \]
\[ F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \phi_i \cos \phi_j}{\pi r^2} dA_j dA_i \]

- Numerical calculation is not easy!

- \[ \sum_{k=1}^{n} F_{ik} = 1 \ (i = 1, \ldots, n) \]

\[ \rightarrow \text{convex surface: } F_{ii} = 0 \text{ (radiation leaving the surface cannot strike itself)} \]

\[ \rightarrow \text{concave surface: } F_{ii} \neq 0 \text{ (possible strike itself)} \]