An Analysis of Opportunistic Forwarding for Correlated Wireless Channels

Anand Seetharam\(^1\), Jim Kurose\(^2\)
\(^1\)Computer Science and Information Technology Program, California State University Monterey Bay
\(^2\)School of Computer Science, University of Massachusetts Amherst
aseetharam@csumb.edu, kurose@cs.umass.edu

Abstract—A variety of forwarding strategies have been developed for multi-hop wireless networks, considering the broadcast nature of the wireless medium and the presence of fading channels that result in time-varying and unreliable transmission quality. One such strategy is opportunistic forwarding, which exploits relay diversity by opportunistically selecting an overhearing relay as a forwarder. Prior work has studied the performance of opportunistic forwarding for the simplified scenario of uncorrelated wireless channels. In this paper, we consider a more realistic scenario of temporally correlated wireless channels; the wireless channel is modeled as a Rayleigh fading channel and its temporal correlation as a modified Bessel function of the first kind and zeroth order. We use these models to develop a simple Markovian model to analyze the performance of opportunistic forwarding for correlated wireless channels for the case of linear networks. We then demonstrate via numerical evaluation the diminishing performance of opportunistic forwarding with increasing channel correlation.

I. INTRODUCTION

The broadcast nature of wireless communication allows for a much richer variety of approaches for forwarding packets between a source and destination than traditional hop-by-hop forwarding along pre-specified paths. In particular, multiple nodes (in addition to the intended next-hop recipient) can overhear transmissions in a wireless network and serve as relays to assist forwarding. Recently, opportunistic forwarding has emerged as a powerful technique for increasing throughput in a wireless network by exploiting the broadcast nature of the medium. Informally, in an opportunistic approach, among the several nodes which receive a copy of a packet transmission, the node that is best able to forward the packet downstream towards its destination takes responsibility for the next transmission of that packet.

Although opportunistic forwarding is well-known in literature, studies analyzing its performance in a network setting are rather limited \cite{1, 2, 3}. Additionally, most prior work including ours \cite{4} study the performance of opportunistic forwarding for the simplified case of uncorrelated wireless channels. In this paper, we investigate the performance of idealized and representative opportunistic forwarding strategies (described in Section III) for the case of correlated wireless channels. Our goal here is not to propose a new opportunistic forwarding protocol, but to investigate the performance of a representative opportunistic forwarding protocols in a realistic setting for the case of temporally correlated wireless channels. Our contributions are as follows:

1) We develop a Markovian model to determine the throughput of opportunistic forwarding for a simple linear network supporting a single flow (e.g., a wireless network along a road) for uncorrelated and temporally correlated wireless channels (Section IV).

2) We demonstrate via numerical evaluation that opportunistic forwarding achieves lower throughput as channel correlation increases (Section V). In case of correlated channels, opportunistic forwarding fails to take advantage of good channel conditions and transmits multiple times over a bad channel resulting in poorer performance than the uncorrelated case (where the channel conditions are independent across time slots).

Our initial exploration shows the need for considering realistic correlated wireless channel models when analyzing wireless network protocols because the performance of these protocols over correlated channels can be significantly different from the uncorrelated channel scenario. Specifically, our work demonstrates the decreasing throughput of opportunistic forwarding for temporally correlated wireless channels.

II. NETWORK MODEL

Let us consider a \(n\)-hop linear network, where \(s\) is the source, \(t\) is the destination, and \(s = r_0, r_1, ..., r_{n-1}, r_n = t\) are the relays (Figure 1). For simplicity, we assume that the distance between \(r_{i-1}\) and \(r_i\) (\(\forall\ i = 1\) to \(n\)) is \(d\) (all analysis in this paper can be easily extended to the case of unequal distance between successive nodes). Let us consider a single transmission between node \(i\) and node \(j\) (\(\forall\ i, j = 1\) to \(n, i \neq j\)). Denote by \(S_{i,j}\) the signal-to-noise ratio from transmitter \(i\) to receiver \(j\):

\[
S_{i,j} = \frac{|x_{i,j}|^2Pd^{-\alpha}}{N_0}
\]

(1)

where \(N_0\) is a constant background noise, \(\alpha\) is the path loss exponent, \(P\) is the transmission power at \(i\) and \(|x_{i,j}|^2\) is the Rayleigh fading coefficient (the flat fading channel is modeled as a Gaussian random process \(x_{i,j}\) \cite{5}). \(|x_{i,j}|^2\) is a random variable, assumed to be exponentially distributed with normalized mean 1. Note that \(d \geq 1\) and \(\alpha \geq 2\).
We consider the simple case of a single flow in the network between $s$ and $t$. We assume a time-slotted system and consider that there is always only a single packet in transit between the source and the destination (i.e., the source transmits a packet and only after it reaches the destination, it starts transmitting another packet). We next present the packet reception probabilities for uncorrelated and temporally correlated fading channels; a packet transmission from node $i$ is said to be successfully received by node $j$ if $S_i \beta_j$ is greater than some threshold $\beta$. We do not consider spatial correlation in this paper.

**Uncorrelated fading channels:** In this case, the fading is assumed to be i.i.d among time slots. This assumption is likely to hold true for fast fading channels where the coherence time is smaller than the duration of a time slot. The packet reception probability ($p$) for a transmission over one hop is given by $p = \exp\left(-\frac{\beta S_i}{P_{nd}}\right)$. Hence, the probability of a successful packet transmission over $n$ hops is given by:

$$p_n = \exp\left(-\frac{\beta N_0}{P_{(nd)}^{\alpha}}\right) = p^{n\alpha}$$

(2)

**Correlated fading channels:** In case of slow fading, the coherence time for the channel is large and hence the channel is likely to be correlated between time slots. Following [5], [6], the fading correlation is modeled in a standard fashion as a modified Bessel function of the first kind and zeroth order. The authors in [5], [6] show using information theory that a Markovian model for a block error process is a good approximation when fading correlation is taken in account, i.e., the success/failure of a transmission between a pair of nodes in a particular time slot is only dependent on the success/failure of the transmission between them in the previous time slot. We denote the parameters of this Markov chain as $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $c$ denotes the $P[success|failure]$. The authors calculate expressions for $a$, $b$, $c$, $d$ in [5], [6]. (Equation 47 in [6]).

### III. Opportunistic Forwarding

In this section, we briefly describe the opportunistic forwarding policies (namely, the greedy opportunistic and the fully opportunistic policies) that we investigate in this paper.

**Greedy opportunistic forwarding:** In this case, a myopic view is adopted. Among the nodes that have a copy of the packet, the node closest to the destination is always selected to transmit the packet. The forwarding strategy begins with the source transmitting a packet. If the source fails to reach the destination or any of the relays, the source retransmits the packet in the next time slot. However if some of the relays receive the packet and the destination does not, then the relay closest to the destination is selected to transmit the packet until it is received at the destination. When the transmission successfully receives the packet, then the next time slot begins with the source transmitting a new packet.

**Fully opportunistic forwarding:** Here we assume the presence of an oracle that is aware of the prevalent channel conditions between all pairs of nodes at the beginning of any time slot. The oracle chooses those transmissions to occur which can move the system to a state where largest number of relay nodes (including the destination) have a copy of the packet for the current channel conditions.

Note the difference between the greedy and the fully opportunistic scenarios: in the greedy scenario, among the nodes possessing the packet, the one closest to the destination will be always given the opportunity to transmit the packet while in the fully opportunistic scenario, the oracle selects the best transmissions to occur after determining channel conditions between all pairs of nodes. Formally, this amounts to saying that for determining the reception probability at a node (say $R$), for the greedy opportunistic case, one has to consider the probability of reaching $R$ from that upstream node closest to the destination possessing a copy of the packet, while for the fully opportunistic case, one has to consider the probability of reaching $R$ from any of the nodes having the packet. Note that the fully opportunistic case provides an upper bound on the throughput achievable by any opportunistic forwarding protocol.

### IV. Markovian Model for Opportunistic Forwarding

In this section, we describe our Markovian model for greedy opportunistic forwarding and present its transition matrix for a simple 3-hop linear network (similar to Figure 1) for uncorrelated and temporally correlated wireless channels. We consider the simple 3-hop network for ease of explanation. Our analytical model can be easily extended to an $n$-hop linear network.

Due to lack of space, we do not present the Markovian model and the corresponding transition matrix for the fully opportunistic case. An approach similar to the greedy opportunistic case can be used to determine the parameters of the transition matrix of the markov chain for the fully opportunistic case. We however present numerical results for both greedy and fully opportunistic cases in Section V.

**Uncorrelated fading channel:** For the case that fading is i.i.d. among the different time slots, we can model greedy opportunistic forwarding using a Markov chain with the following state space, namely $A = \{1, 0, 0, 0\}$, $B = \{0, 1, 0, 0\}$ and $C = \{0, 0, 1, 0\}$, where 1 denotes that a node has a packet and 0 denotes that the node does not have a packet. We do not need a fourth state denoting that the packet is received at destination $t$ because in the next time slot, $s$ will transmit a new packet and hence one can assume a transition to state $A$ if $t$ receives the packet. The transition matrix $\{M_u\}$ for this Markov chain is given below.

$$M_u = \begin{pmatrix} p_3 + q_1q_2q_3 & p_1q_2q_3 & p_2q_3 \\ p_2 & q_1 & p_1 \\ p_1 & 0 & q_1 \end{pmatrix}$$

(3)

where $p_i$ and $q_i$ are the success and failure probabilities of a transmission over $i$ hops respectively. If the steady state probabilities for states $A$, $B$ and $C$ are denoted by $\pi_A$, $\pi_B$ and $\pi_C$, then the throughput of the system under the greedy opportunistic policy is given by,

$$T = p_3\pi_A + p_2\pi_B + p_1\pi_C$$

(4)

Note that if there are $(n+1)$ nodes in the network the state space will be $n$. 
Correlated fading channel:

When we relax the i.i.d. assumption, each state will now be split into two states. Let us consider state $A$. We split $A$ into two states $A_1 = \{0,0,0\}$ and $A_2 = \{1^*,0,0\}$. Each node can effectively be in three states:

- 0 denotes that a node does not have a packet.
- 1 denotes that a node is going to transmit a packet for the first time, i.e., it will receive an independent fade to all nodes downstream.
- $1^*$ denotes that a node transmitted in the previous time slot and none of the nodes downstream received the packet, i.e., it has a bad fade to all nodes downstream.

This simple classification is sufficient; the states of the Markov Chain are $A_1 = \{0,0,0\}$, $A_2 = \{1^*,0,0\}$, $B_1 = \{0,1,0,0\}$, $B_2 = \{0,1^*,0,0\}$, $C_1 = \{0,0,1,0\}$, $C_2 = \{0,0,1^*,0\}$. Note that if there are $(n+1)$ network nodes, the state space will be $2n$. The transition matrix $\{M_c\}$ for the four state Markov chain is shown in (5). Here $c_i$ denotes the probability of successfully transmitting a packet to a node $i$ hops away (considering correlated fading), given that the previous transmission to that node was a failure. $p_i$ and $q_i$ denote the success and failure probabilities of a transmission over $i$ hops respectively (considering uncorrelated fading). Considering $\pi$ as the steady state distribution of the Markov chain, the throughput $T$ is given by

$$T = p_3\pi_{A_1} + c_3\pi_{A_2} + p_2\pi_{B_1} + c_2\pi_{B_2} + p_1\pi_{C_1} + c_1\pi_{C_2}$$

V. NUMERICAL EVALUATION

We present numerical results comparing the throughput of greedy and fully opportunistic forwarding for the uncorrelated and temporally correlated fading scenarios (Figure 2). As expected the throughput of fully opportunistic forwarding is higher than greedy opportunistic forwarding. In Figure 2, $\rho_{ho}$ denotes the coefficient of correlation between successive fading samples. We observe from the figure that the throughput of opportunistic forwarding (greedy/fully) for correlated fading is lower than that for the corresponding uncorrelated fading.

This result corresponds with intuition and we explain it for greedy opportunistic forwarding (consider the red, pink and blue lines). In greedy opportunistic forwarding, it is always the case that among the nodes which have a copy of a packet, the one closest to the destination transmits the packet. If a transmitting node (say $s$) has a bad channel to nodes closer to the destination than itself, then the transmission will be unsuccessful and $s$ will have to retransmit the packet. In case of correlated fading (unlike the uncorrelated channel case), the channel is likely to remain bad in future as well, with the effect that future transmissions will also be unsuccessful. When $s$ has a good channel to the nodes closer to the destination than itself, the transmission is likely to be successful. But in the next time slot, $s$ will not get the chance to transmit again as some other node closer to the destination will transmit the packet. As greedy opportunistic forwarding fails to take advantage of good channel conditions, its performance is lower for correlated fading in comparison to uncorrelated fading.

VI. CONCLUSION

In this paper, we investigated the performance of opportunistic forwarding for the case of uncorrelated and temporally correlated wireless channels. We constructed Markovian models to analytically determine the throughput of greedy and fully opportunistic forwarding for linear wireless networks. Our evaluations showed that the throughput of opportunistic forwarding is lower for correlated fading channels in comparison to uncorrelated channels. We attribute the reduced performance of opportunistic forwarding for correlated channels to the fact that while relay nodes fail to take advantage of good channel conditions, they transmit multiple times over bad channels.

REFERENCES