Viewing Transformation

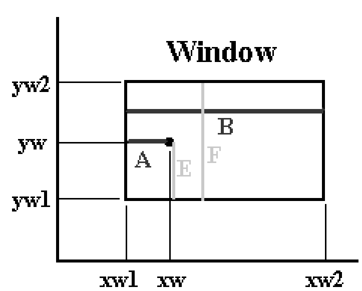
Clipping

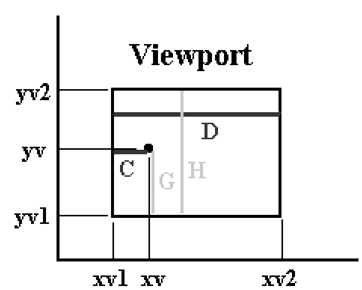
2-D Viewing Transformation

2-D Viewing Transformation

- Convert from Window Coordinates to Viewport Coordinates
- (xw, yw) --> (xv, yv)
- Maps a world coordinate window to a screen coordinate viewport
- Window defined by: (xw1,yw1), (xw2,yw2)
- Viewport defined by: (xv1,yv1), (xv2,yv2)
- Basic idea is to maintain proportionality

Window to Viewport Transformation





A/B = C/D

$$\frac{xw-xw1}{xw2-xw1} = \frac{xv-xv1}{xv2-xv1}$$

$$\boxed{xv=(Wv/Ww)*xw + xv1-(Wv/Ww)*xw1}$$

where:
Ww=xw2-xw1 (window width)
Wv=xv2-xv1 (viewport width)

E/F = G/H

$$\frac{yw-yw1}{yw2-yw1} = \frac{yv-yv1}{yv2-yv1}$$

$$\boxed{yv=(Hv/Hw)*yw+ yv1-(Hv/Hw)*yw1}$$

where:
Hw=yw2-yw1 (window height)
Hv=yv2-yv1 (viewport height)

Viewing Transformation in Windows: Mapping Modes

Windows Viewing Transformation: Mapping Modes

- Create logical coordinate system
  - Define direction of axes
  - Define units
  - Can also move the origin
- Windows maps output to real device
  - e.g., plot at 100,100 "logical millimeters”
  - Windows figures out where on screen
  - Not exact, but close
- It’s Windows way of implementing the viewing transformation

Windows Mapping Modes

MAPPING MODE	LOGICAL UNIT	X-AXIS	Y_AXIS
MM_TEXT	Pixel	Right	Down
MM_HIENGLISH	.001 inch	Right	Up
MM_LOENGLISH	.01 inch	Right	Up
MM_HIMETRIC	.01 mm	Right	Up
MM_LOMETRIC	.1 mm	Right	Up
MM_TWIPS	1/20 point=1/1440″	Right	Up
MM_ISOTROPIC	Arbitrary (x==y)	Selectable	
MM_ANISOTROPIC	Arbitrary (x!=y)	Selectable	

Changing the Mapping Mode

- pDC->SetMapMode(MAP_MODE);
- Maps logical coordinates to device coordinates
  - Device Coordinate (physical)
    - units: pixels
    - +x: right, +y: down
  - Converts logical ("window") to device ("viewport") coordinates as follows

    xV = (xVExt/xWExt) * (xW - xWOrg) + xVOrg
    yV = (yVExt/yWext) * (yW - yWOrg) + yVOrg
- (xWOrg,yWOrg) and (xVOrg,yVOrg) are the origins of the window and viewport
- Both are (0,0) in the default device context

Moving Origins

- pDC->SetWindowOrg(x,y); // logical units
  - For x,y positive, think of this as moving the upper left-hand corner of the physical device viewport (screen) up and right by (x,y) logical units
- pDC->SetVieportOrg(x,y); // device units--pixels
  - For x,y positive, think of this as moving the lower left-hand corner of the logical window down and right by (x,y) device units
- Both move the coordinate system origin to (x,y), but units of x,y are different

Variable Unit Mapping Modes

- Coordinate axes can have any size/orientation
- MM_ISOTROPIC -- x & y units must be same size
- MM_ANISOTROPIC -- different x and y units
- Set the X and Y scaling factors with:

  pDC->SetWindowExt (xWExt, yWExt);

  pDC->SetViewportExt (xVExt, yVExt);
- X scaling factor in going from Logical Coordinates to Device Coordinates = xVExt/xWExt
- Y scaling factor = yVExt/yWExt

Example 1

- Create coordinate system where each logical unit is two pixels:
  – twice the default device unit coordinates

```
pDC->SetMapMode (MM_ISOTROPIC);
pDC->SetWindowExt (1, 1);
pDC->SetViewportExt (2, 2);
```

Example 2

- Create coordinate system with y-axis up, each y-unit = 1/4 pixel; x-axis unchanged:

```
pDC->SetMapMode (MM_ANISOTROPIC);
pDC->SetWindowExt (1, -4);
pDC->SetViewportExt (1, 1);
```

Example 3

- Create coord system where client area is always 1000 units high & wide, y-axis up:

```
CSize size;
size = pDC->GetWindowExt (); // get client area size
   // returns size in default device units--here pixels
pDC->SetMapMode (MM_ANISOTROPIC);
pDC->SetWindowExt (1000, -1000);
pDC->SetViewportExt (size.cx, size.cy);
```

- Now (1000,1000) will always be at upper right edge of client area

OpenGL Viewing Transformation

- OpenGL designed for 3D graphics
- Must project onto 2D window
- Also do window to viewport transformation
  - with clipping
- For 2D graphics, use an orthographic projection
  - gluOrtho2D(xmin,xmax,ymin,ymax)
    - Equivalent to taking z=0 & setting a "window" with clipping boundaries: xmin<=x<=xmax, ymin<=y<=ymax -- logical units used
      - Will be mapped to entire client area of physical window
      - Client area determined by:
        - glutInitWindowSize(width,height)
        - Device units used

OpenGL Viewport

- gluOrtho2d(left,right,bottom,top) and glutInitWindowSize(w,h) map the “window” to the entire w X h client area
- glViewport(x,y,w,h) maps the “window” to the specified viewport within the client area
  - Device units used

Clipping

Clipping

- Elimination of parts of scene outside a window or viewport
- Clipping with respect to a window (Given: xwmin, ywmin, xwmax, ywmax)
  - Clip at this level ==> fewer points go through viewing transformation
- Clipping with respect to a viewport (Given: xvmin, yvmin, xvmax, yvmax)

Clipping

- Points
- Lines
  - Cohen-Sutherland Line Clipper
- Polygons
  - Sutherland-Hodgeman Polygon Clipper
  - Weiler-Atherton Polygon Clipper
- Other Curves
- Text

Point Clipping

- Given:
  - point (x,y)
  - clipping rectangle (window or viewport) (xmin,ymin,xmax,ymax)
- Point test:

  if ((x<=xmax) && (x>=xmin)

  && (y<=ymax) && (y>=ymin)

  the point x,y lies inside the clip area
  - so keep it!

Line Clipping

- Could apply point test to all points on the line
  - Too much work
- Need a simple test involving the line's endpoint coordinates

Cohen-Sutherland Line Clipper

- Observation-- All lines fall into one of three categories
  1. Both endpoints inside clip rectangle
     - (Trivially accept entire line)
  2. Both endpoints outside clip rectangle on the same side of one of its borders
     - (Trivially reject entire line)
  3. Neither 1 nor 2
     - (Chop off part of line outside one of borders and repeat)

Region Code

- A tool in assigning lines to Category 1 or 2
- 4-bit region code number assigned to an endpoint (x,y)
- Any set bit means endpoint is outside of one of the 4 borders of the clip rectangle
- Each bit position corresponds to a different border

Region Code RC = LRBT

- L=left (if x<xmin, L=1, else L=0)
- R=Right (if x>xmax, R=1, else R=0)
- B=Bottom (if y<ymin, B=1, else B=0)
- T=Top (if y>ymax, T=1, else B=0)
- The Region Code Divides the entire x-y plane 9 regions

Region Codes (LRBT)

```
            |        |
   1001     |  0001  |   0101
            |        |
 ------------------------------
            |        |
   1000     |  0000  |   0100
            |        |
 ------------------------------
            |        |
   1010     |  0010  |   0110
            |        |
```

Category 1 Lines

- Assume region codes for the line's endpoints are RC1 and RC2
- Take Boolean OR of two region codes

  if (RC1 | RC2 == 0)

  both RCs are 0000

  both endpoints are inside

  so it's Category 1 (trivial accept)

Category 2 Lines

- Both endpoints are outside same border
  - (Category 2 line)
- Then both region codes will have the same bit set in one of the four bit positions
  - Boolean AND will give a non-zero result:

    if (RC1 & RC2 != 0)

    - both endpoints are outside same border
    - so it's Category 2 (trivial reject)

Category 3 Lines

- Want to chop off outside part of line
- May have both endpoints (P1 & P2) outside different borders of clip region
  - So it's not important which end is chopped off first
- But if one endpoint's in and other's out:
  - Want to chop off the outside end
  - So Arrange things so P1 is the outside point
    - (swap P1 & P2 if necessary)

How to do the Chopping

- Want to determine the new endpoint
- Endpoint coordinates (x1,y1), (x2,y2) are known
- Slope m can be computed from them
- So y = m*(x-x2) + y2 (point slope form)
- Or x = (y-y2)/m + x2
- Look at P1's region code (RC1)
- Four possible cases:

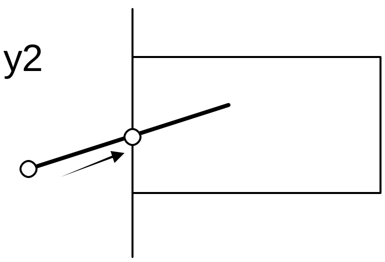
If RC1 == 1xxx (P1 to left of xmin)

- New endpoint should be on the left boundary:

  x1 <-- xmin

  y1 <-- m*(xmin-x2) + y2

  Reset RC's L bit



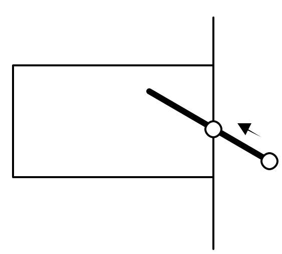
If RC1 == x1xx (P1 right of xmax)

- New endpoint should be on the right boundary:

  x1 <---xmax

  y1 <---m*(xmax-x2) +y2

  Reset RC's R bit



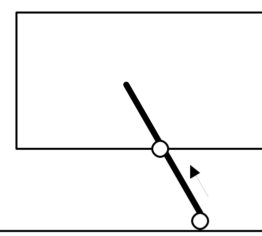
If RC1 == xx1x (P1 below ymin)

- New endpoint should be on the bottom boundary:

  y1 <---ymin

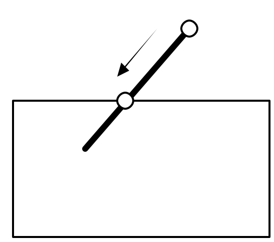
  x1 <---(ymin-y2)/m + x2

  Reset RC's B bit



If RC == xxx1 (P1 above ymax)

- New endpoint should be on the top boundary:
  - y1 <---ymax
  - x1 <---(ymax-y2)/m + y2
  - Reset RC's T bit



- Horizontal and vertical lines are special cases
  - Horizontal:
    - y doesn't change and x = xboundary
  - Vertical:
    - x doexn't change and y = yboundary

The C-S Line Clipping Algorithm

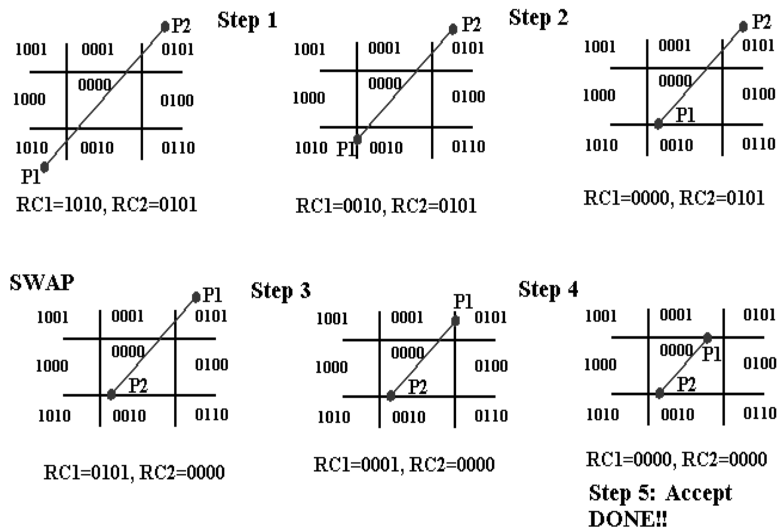
- Input:
  - Original endpoints (x1,y1,x2,y2)
  - Clip region boundaries (xmin,ymin,xmax,ymax)
- Output:
  - Accept Code (AC)
    - AC==TRUE ==> some part of line was inside
    - AC==FALSE ==> no part of line was inside
  - Clipped Line endpoints (x1,y1,x2,y2)
    - only if AC==TRUE

C-S Algorithm Pseudo-code:

```
CS_LineClip(xmin,ymin,xmax,ymax,x1,y1,x2,y2,AC)
   done = FALSE
   While (!done)
      Calculate endpoint codes rc1, rc2
      If ((rc1 | rc2) == 0 )    // Category 1
         done = TRUE
         AC = TRUE
      Else
         If ((rc1 & rc2) != 0)  // Category 2
            done = TRUE
            AC = FALSE
         Else
            If (P1 is inside)
              Swap (x1,y1), (x2,y2); and rc1,rc2
```

```
            If (L-bit of rc1 is set)     // 1xxx
              x1 = xmin
              y1 = m*(xmin-x2) + y2
            Else
              If (R-bit of rc1 is set)   // x1xx
                x1 = xmax
                y1 = m*(xmax-x2) + y2
              Else
                If (B-bit of rc1 is set) // xx1x
                  y1 = ymin
                  x1 = (ymin-y2)/m + x2
                Else                     // xxx1
                    y1 = ymax
                    x1 = (ymax-y2)/m + x2
```

Cohen-Sutherland Clipping Example



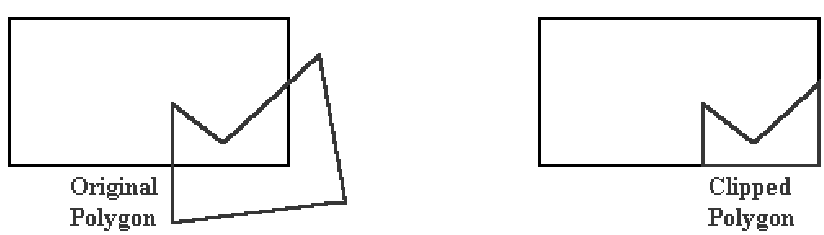
Polygon Clipping

Polygon Clipping

- Clip a polygon to a rectangular clip area
- Input
  - Ordered list of polygon vertices (nin, vin[ ])
  - Clip rectangle boundary coordinates (xmin, ymin, xmax, ymax).
- Output:
  - An ordered list of clipped polygon vertices (nout, vout[ ]).
  - vin[ ] and vout[ ] could be arrays of POINTs

Approaches to Polygon Clipping

- Use a line clipper on each polygon edge???
- But we usually won't get back a polygon
  - Parts of the clip rectangle will be edges of the clipped polygon that line clipper won't get
- Really need new list of edges (or vertices)



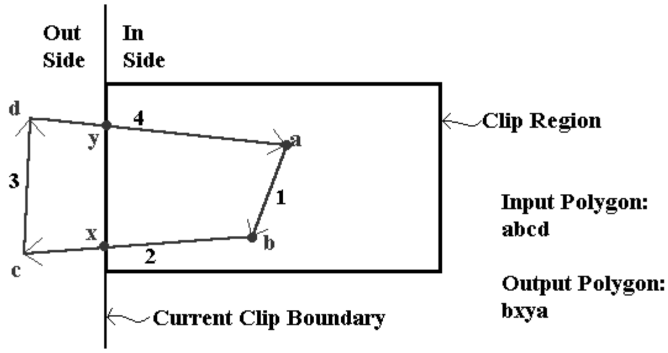
Sutherland-Hodgeman Polygon Clipper

- Approach:
  - Clip all polygon edges with respect to each clipping boundary
  - Do four passes; on each pass:
    - Traverse current polygon and clip with respect to one of the four boundaries
    - Assemble output polygon edges as you go
    - vin[ ] --> Clip Left --> vtemp1[ ] --> Clip Right --> vtemp2[ ] --> Clip Bottom --> vtemp3[ ] --> Clip Top --> vout[ ]

- On any polygon traversal the clip boundary divides plane into "in" side and "out" side
- For any given edge (vertices i and i+1),
  - during traversal, there are four possibilities:
  - (Assume vertex i has already been processed)

VERTEX i	VERTEX i+1	ACTION
in	in	Add Vertex i+1 to output list
out	out	Add no vertex to output list
in	out	Add intersection point with edge to output list
out	in	Add intersection point with edge and vertex i+1 to output list

Sample Traversal



Traversal		Type	Action
1	a → b	in-in	Add point b
2	b → c	in-out	Add intersection point x
3	c → d	out-out	Add nothing
4	d → a	out-in	Add intersection point y and point a

Implementation

- Function sh_clip()
  - Will clip an input polygon (ni, vi[ ])
  - With respect to a given boundary (bndry)
  - Generating an output polygon (no, vo[ ])
- Enumerate the boundaries as:
  - LEFT, RIGHT, BOTTOM, and TOP

sh_clip(ni, vi[ ], no, vo[ ], xmin, ymin, xmax, ymax, bndry);

vi[ ] and vo[ ]: could be arrays of POINTs

ni, no: number of points in each array

xmin, ymin, xmax, ymax: clip region boundaries

Using sh_clip() to clip a polygon

- Make four calls to sh_clip():

```
sh_clip(nin, vin[ ], ntemp1,  vtemp1[ ], xmin, ymin,
          xmax, ymax, LEFT);
sh_clip(ntemp1, vtemp1[ ], ntemp2, vtemp2[ ], xmin,
          ymin, xmax, ymax, RIGHT);
sh_clip(ntemp2, vtemp2[ ], ntemp3, vtemp3[ ], xmin,
          ymin, xmax, ymax, BOTTOM);
sh_clip(ntemp3, vtemp3[ ], nout, vout[ ], xmin, ymin,
          xmax, ymax, TOP);
```

Three Helper Functions

BOOL inside(V, xmin, ymin, xmax, ymax, Bndry)

- Returns TRUE if vertex point V is on the "in" side of boundary Bndry

intersect(V1, V2, xmin, ymin, xmax, ymax, Bndry, Vnew)

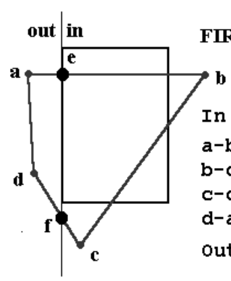
- Computes intersection point of edge whose endpoints are V1 and V2 with boundary Bndry
- Returns the resulting point in Vnew

output(V, n, vout[ ])

- Adds vertex point V to the polygon (n, v[ ])
  - n will be incremented by 1
  - vertex V added to end of polygon's vertex list v[ ]

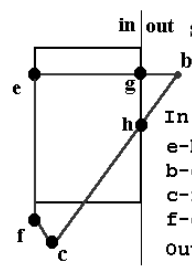
```
sh_clip (ni, vi[], no, vo[], bndry)
no = 0                          // output list begins empty
First_V = vi[0]                 // first vertex (i)
For (j=0 to ni-1)               // traverse polygon
  Second_V = v[(j+1) % ni]      // second vertex (i+1)
  If (inside(First_V, bndry)
    If (inside(Second_V, bndry)  // "in-in" case
       output(Second_V, no, vo)
    Else                         // "in-out" case
       intersect(First_V, Second_V, bndry, Vtemp)
       output (Vtemp, no, vo)
   Else
    If (inside(Second_V, bndry)  // "out-in" case
       intersect(First_V, Second_V, bndry, Vtemp)
       output(Vtemp, no, vo)
       output(Second_V, no, vo)   // no "out-out" case
  First_V = Second_V             // prepare for next edge
```

Example of S-H Clipping



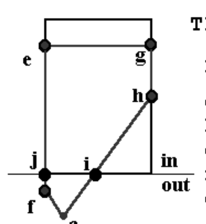
FIRST TRAVERSAL

In polygon: abcd
a-b (o-i) → e,b
b-c (i-i) → c
c-d (i-o) → f
d-a (o-o) → none
Out polygon: ebcf



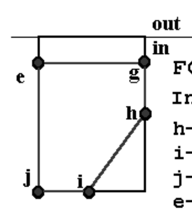
SECOND TRAVERSAL

In polygon: ebfc
e-b (i-o) → g
b-c (o-i) → h,c
c-f (i-i) → f
f-e (i-i) → e
Out polygon: ghcfe



THIRD TRAVERSAL

In polygon: ghcfe
g-h (i-i) → h
h-c (i-o) → i
c-f (o-o) → none
f-e (o-i) → j,e
e-g (i-i) → g
Out polygon: hijeg

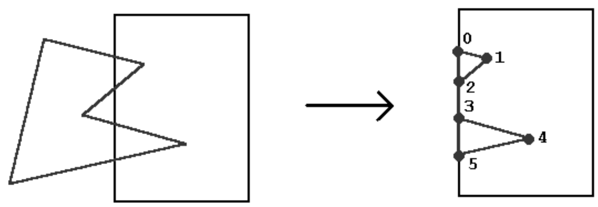


FOURTH TRAVERSAL

In polygon: hijeg
h-i (i-i) → i
i-j (i-i) → j
j-e (i-i) → e
e-g (i-i) → g
g-h (i-i) → h
Final Out polygon: ijegh

Sutherland-Hodgeman Problems

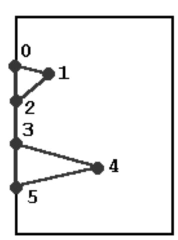
- Works fine with convex polygons
- But some concave polygons problematic
  - Extraneous edges along a clip boundary may be generated as part of the output polygon
  - Could cause problems with polygon filling



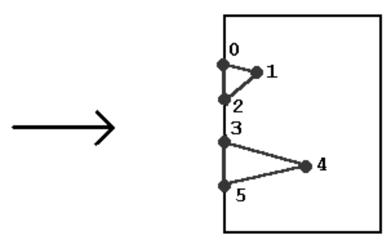

```
Output Polygon: 0,1,2,3,4,5
Extraneous Edge: 2-3
```

Solutions to S-H Problems

- Add a postprocessing step
  - Check output vertex list for multiple (>2) vertex points along any clip boundary
  - Correctly join pairs of vertices




```
Polygon: 0,1,2,3,4,5
0,2,3,5 are vertices
along the left boundary
```




```
So break into two polygons:
0,1,2 and 3,4,5
```

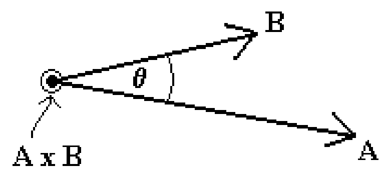
Other Solutions

- Add a preprocessing step
  - Split concave polygon into convex polygons
- Or use a more general clipping algorithm
  - For example, the Weiler-Atherton polygon clipper

Splitting Concave Polygons

- Split into convex polygons
- Use edge vector cross products

Vector Product of Two Vectors



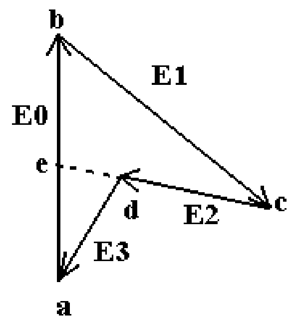
- V = A X B
- |V| = |A| |B| sin(?)
- Direction: RH Rule
- In terms of components

```
             | i  j  k  |
A X B  =     | Ax Ay Az |
             | Bx By Bz |
i, j, k: unit vectors in x, y, z directions
```

Splitting Concave Polygons

- Process edges in clockwise order
- Form successive edge vectors
- Compute vector cross product between successive edge vectors
- If all cross products are not negative
  - Polygon is concave
  - Split it along line of first edge vector in the cross-product pair:
    - Compute intersections of this line with other edges
    - This splits polygon into two pieces
- Repeat this until no other edge cross products are positive

Splitting Concave Polygons



abcd --> aed & ebc

E0 X E1 --> -k
E1 X E2 --> -k
E2 X E3 --> +k ==> find int. pt.
E3 X E0 --> -k

$$E0 \times E1 = \begin{vmatrix} i & j & k \\ Dx0 & Dy0 & 0 \\ Dx1 & Dy1 & 0 \end{vmatrix}$$

Dx0 = xb-xa
Dy0 = yb-ya

Splitting Convex Polygon into Triangles

- Often convenient since triangles are the simplest polygon
  1. Define a sequence of three consecutive vertices to be a new polygon (triangle)
  2. Delete middle vertex from original vertex list
  3. Continue to form triangles until original polygon has only three vertices

Weiler-Atherton Polygon Clipper

- Clips a "Subject Polygon" to a "Clip Polygon"
- Both polygons can be of any shape
- Result: one or more output polygons that lie entirely inside the clip polygon
- Basic idea:
  - Follow a path that may be a subject polygon edge or a clip polygon boundary

The Weiler-Atherton Algorithm

1. Set up vertex lists for <u>subject</u> and <u>clip</u> polygons

   Ordering: as you move down each list, inside of polygon is always on the right side (clockwise)
2. Compute all intersection points between subject polygon and clip polygon edges

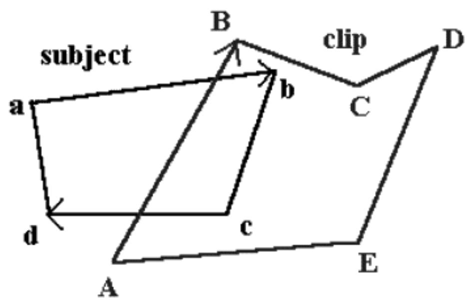
   Insert them into each polygon's list

   Mark as intersection points

   Mark “out-in” intersection points

   (subject polygon edge moving from outside to inside of clip polygon edge)

Intersection Points & out-in Marking (General)



$\overline{ab}$:
x = xa + (xb-xa)t
y = ya + (yb-ya)t

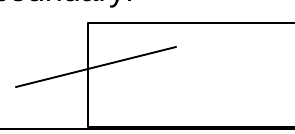
$\overline{AB}$:
x = xA + (xB-xA)s
y = yA + (yB-yA)s

Solve for s and t
0<=t<=1 and 0<=s<=1 ==>
Intersection Point

Vector cross product:
$\overline{ab}$ X $\overline{AB}$ = +k ==> Out-In
$\overline{cd}$ X $\overline{AB}$ = -k ==> In-Out

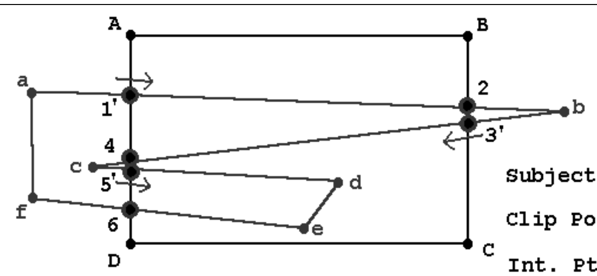
Intersection Points and Out-In Marking (Simple)

- If clip polygon is a rectangle:
  - Use point in/out test
  - e.g., for intersection with left boundary:
    x<xmin means outside, x>=xmin means inside
- Intersections also easy
  - Use Cohen-Sutherland ideas
    - e.g., for intersection with left boundary:
      x = xmin
      y = m*(xmin-x1) + y1



Weiler-Atherton Algorithm, continued

3. Do until all intersection points have been visited:
   - Traverse subject polygon list until a non-visited out-in intersection point is found;
   - Output it to new output polygon list
   - Make subject polygon list be the active list
   - Do until a vertex is revisited:
     - Get next vertex from active list & output
     - If vertex is an intersection point,
       - make the other list active
   - End current output polygon list



Subject Polygon: a-b-c-d-e-f

Clip Polygon: A-B-C-D

Int. Pts.: 1,2,3,4,5,6

1st Iteration:

Subject: a → 1' → 2   b   3' → 4   c   5'   d   e   6   f

Clip: A   B   2 → 3'   C   D   6   5'   4 → 1'

1 revisited so stop out poly

Output: 1   2   3   4

2nd Iteration:

Subject: a   1'   2   b   3'   4   c → 5' → d → e → 6   f

Clip: A   B   2 → 3'   C   D   6 → 5'   4   1'

5 revisited so stop out poly

Output: 5   d   e   6

All intersection points visited ⇒ Done!

Clipping Other Curves

- Must compute intersection points between curve and clip boundaries
- In general solve nonlinear equations
- Many times approximation methods must be used
- Time consuming

Clipping Text

- Use successively more expensive tests

  1. Clip string

     Embed string in rectangle

     Clip rectangle (4 point tests)

     • entirely in ==> keep string

     • entirely out==>reject string

     • neither==>next test

2. Clip each Character

   Embed character in rectangle

   Clip rectangle (4 point tests)

   - entirely in ==> keep character
   - entirely out==>reject character
   - neither==>next test

3. Two possibilities for Character Clipping
   - Bitmapped: look at each pixel
   - Stroked: Apply line clipper to each stroke